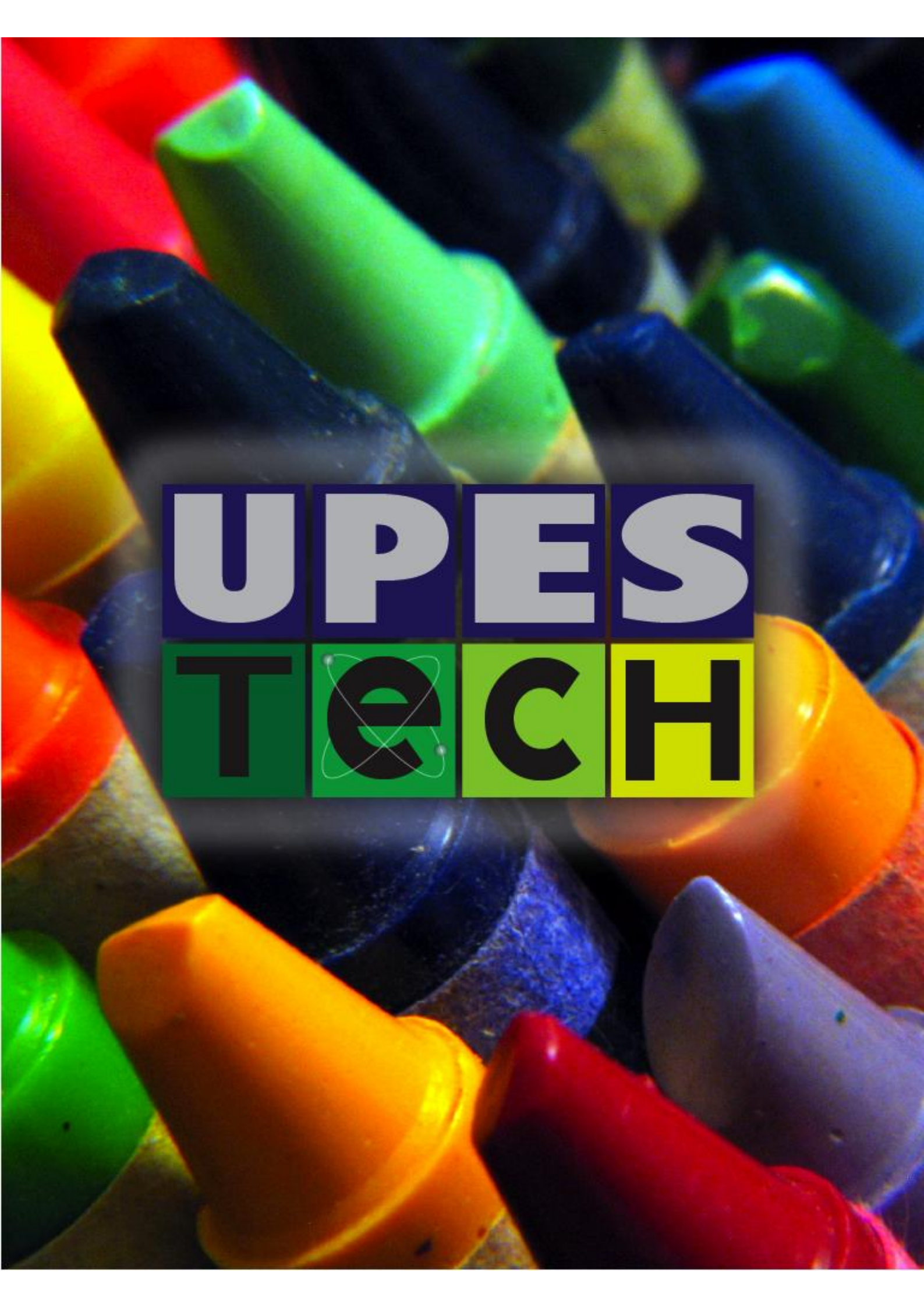




UPES
TECH

Date

Data Structures

Scope Of a Variable

① Static Scoping

② Dynamic Scoping

int a=500, b=600;

Main()

```
{ int a, b;  
  a=10; b=20;  
  swap();  
  printf("%d %d", a, b);  
}
```

swap()

```
{ int t;  
  t=a;  
  a=b;  
  b=t;  
  printf("%d %d", a, b);  
}
```

free variables (means variables are not in this fn.)

O/p:-
600 500
10 20

Static Scoping :-

O/p:- 600 500 10 20

when the free variables are taken from global variables.

Dynamic Scoping :-

O/p:- 20 10 20 10

when the free variables are taken from previous function call till we reach main().

If we don't get variables in even in main, then we refer dynamic scoping global variable.

* All language follow Static Scoping, older languages follow dynamic scoping.

O/p in C:

600 500 10 20

* int a=10, b=20;

main()

```
{ int a=1, b=1;  
  A();  
}
```

A()

```
{ int a=2, b=2;  
  B();  
}
```

B()

```
{ int a=3, b=3;  
  printf("%d %d", a, b);  
}
```


Date

04.08.12

- boolean b = true;
- static scoping: true
- dynamic scoping: false

procedure p

begin

pf(b);

end

begin

boolean b = false;

p;

end

b
true
1000

b
false
2000

no name
for this fn.
∴ it is the main
fn.)

- var x: real
- procedure show()

begin

pf(x);

end

procedure small()

begin

var x: real

x = 0.125;

show();

end

begin

x = 0.25

show();

small();

end

no name
for this fn,
hence main

no concept of stack.

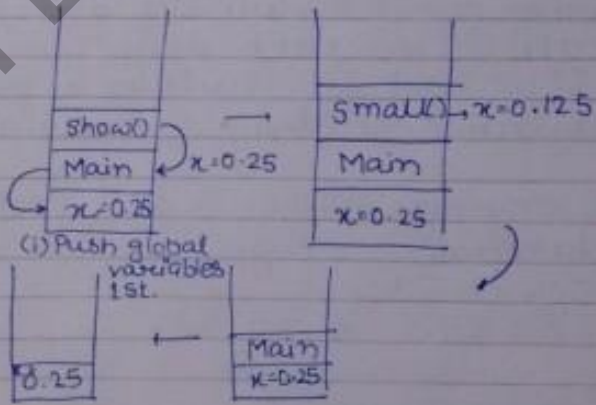
since at compile time, we know
only global variable.
(at compile time)

• static scoping: 0.25 0.25

• dynamic scoping: 0.25 0.125

(at run time)

For dynamic:-



- var x, y: integer
- procedure p (var n: integer)

begin

x = (n+2)/(n-3);

end

• static: 3 6

• dynamic: 6 7

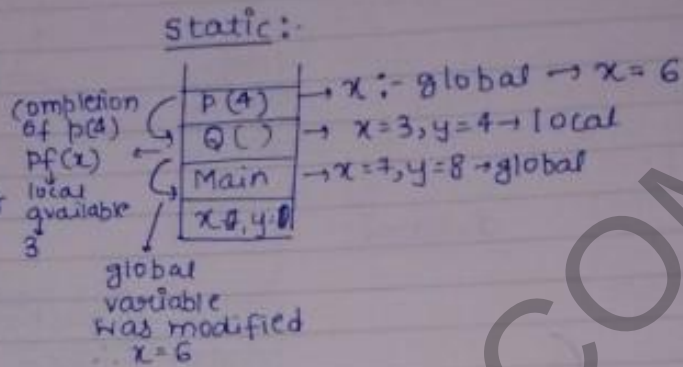
begin

x = 7; y = 8;

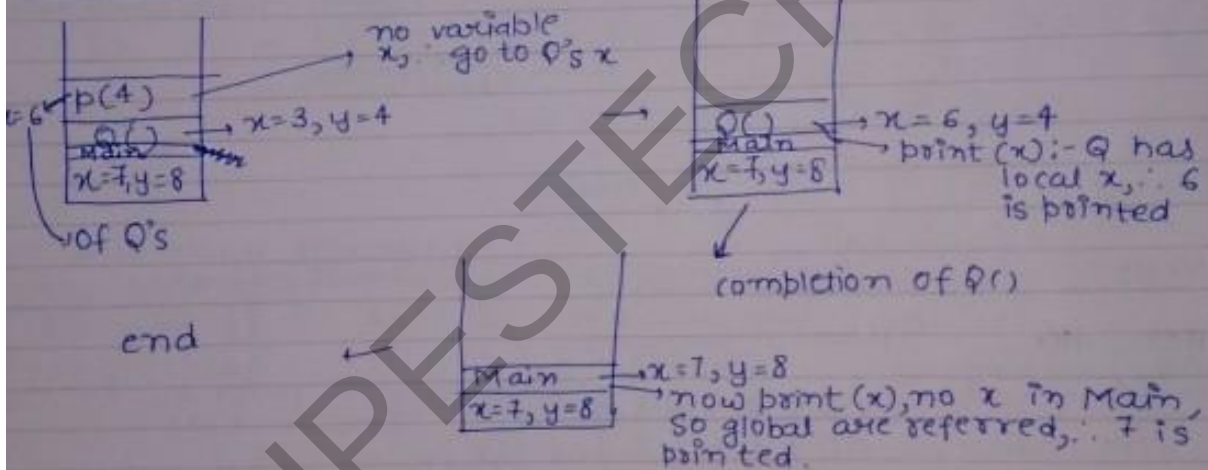
```

Q();
pf(x);
end
procedure Q
begin
  var x,y: integer
  x=3,y=4;
  p(y);
  pf(x);
end

```



dynamic:-



```

Q. var x: integer
procedure two
begin
  pf(x);
end
procedure one
begin
  var x: integer
  x=5;
  two;
end

```



```

begin
  r=2;
  two;
  one;
  two;
end

```

changed by main $\rightarrow$ $\boxed{2}$ $\leftarrow$ global
1000

Static: 2

↑
printed by pfc() of two (global variable)

2

↑
printed by pfc() of two (when called by one)

2

dynamic:

two()
Main
r=2

no r here so goes back to main (rather we say go to global)

two()
one
Main
r=2

so go back to two
no r here $\rightarrow$ r=5

$\therefore$ O/p :- 2 5 2

two()
Main
r=2

two() completed & hence main too

one completed
no r here so goes back to main

• int a; $\rightarrow$

a
1000

big endian

• a=55; $\rightarrow$

0000 0000 0011 0111
1000 1001

1000 1001
0011 0111

little endian

7	6	5	4	3	2	1	0
0	0	1	1	0	1	1	1

55 $\rightarrow$ nearest power of 2 (smaller is 2^5)

$2^5 \rightarrow 32 \rightarrow \begin{array}{r} 55 \\ -32 \\ \hline 23 \end{array}$

23 $\rightarrow$ nearest power of 2 (smaller is 2^4)

$2^4 \rightarrow 16 \rightarrow \begin{array}{r} 23 \\ -16 \\ \hline 7 \end{array}$

7 $\rightarrow$ nearest power of 2 (smaller is 2^2)

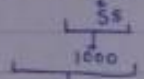
$2^2 \rightarrow 4 \rightarrow \begin{array}{r} 7 \\ -4 \\ \hline 3 \end{array}$

3 $\rightarrow$ nearest power of 2 (smaller is 2^1)

$2^1 \rightarrow 2 \rightarrow \begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array}$

1 $\rightarrow$ nearest power of 2 (smaller or equal)

$2^0 \rightarrow 1 \rightarrow \begin{array}{r} 1 \\ -1 \\ \hline 0 \end{array}$

`pf(a) → 55`
`pf(&a) → 1000`
`pf(*(&a)) → 55`

 value at address 1000

`pf("%d", a) → 55`
`pf("%u", &a) → 1000`
 unsigned
 (for addresses)

`pf("%d", *(&a)) → 55`

• `int a;` → 

• `a = 55;` → 

• `a → 55`

• `b → 1000`

• `*b → 55`

• `*a → error` (compilation error) • `printf("%u", b) → 1000`

• `&b → 2000`

• `*(&b) → 1000`

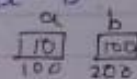
`&` → referencing operator

`*` → dereferencing

`int a = 10, b = -1;`
`printf("%u", a); → 10`

`printf("%u", b); → 2's complement of -1 (if range of int is 0 to 4294967295)`
`printf("%d", b); → -1`
 then the o/p is 4294967295

`int b = &a;` // we can't write like this,
 // because writing `int b;`
 // we committed that we will
 // store a value at 'b'

we write :- `int *b = &a;` 

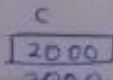

`b`
 1000
 2000

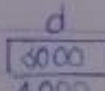
`int a = 10;`
`int b = (int)&a;`
`pf("%d", a) → 10`
`pf("%d", &a) → 1000`
`pf("%d", b) → 1000`
`pf("%d", *b) → compile time error`

`printf("%d", *b) → 55`
`printf("%u", &b) → 2000`

Q1 `int a = 10;` → 

`int *b = &a;` → 
`b`
 1000
 2000

`int **c = &b;` → 
`c`
 2000
 3000
 because b is itself a pointer

`int ***d = &c;` → 
`d`
 3000
 4000
 d is storing an address which is a pointer to a pointer.

• Single pointer :- storing address of a value.

• double pointer :- storing address of a pointer.

- ★ If variable is holding a value, then to get the value → no [★] req.
- ★ " " " " " single pointer, then to get the value → 1 [★] req.
- ★ " " " " " double pointer, " " " " " → 2 [★] req.

Parameter Passing Techniques

1. Call by value / Call by Copy
2. Call by reference
3. Call by copy-restore
4. Call by Name
5. Call by Need
6. Call by Text

Call by Value:-

main()

```
{ int a=10, b=20;
```

```
  printf("/d/d", a, b);
```

```
  swap(a, b);
```

```
  printf("/d/d", a, b);
```

```
}
```

```
swap(int c, int d)
```

```
{ int t;
```

```
  t = c; // c=20
```

```
  c = d; // d=10
```

```
  d = t;
```

```
}
```

• Without using temp. variable.

```
a = a+b;
```

```
b = a-b;
```

```
a = a-b;
```

O/p:-

10 20 10 20

a	b
10	20
1000	2000

c	d	t
20	10	10
3000	1000	5000

★ In C compiler, by default call by value is done,

i.e. swap(a, b) → call by value

★ If any compiler supports call by reference by default

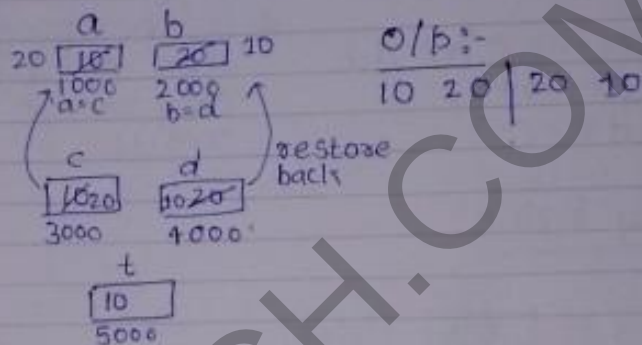
then swap(a, b) → call by reference
(it will pass addresses of a & b, & not values of a & b)

Call by Copy - Restore

```
main()
{
    int a=10, b=20;
    printf("a,b");
    swap(a,b);
    printf("a,b");
}

swap(int c, int d)
{
    int t;
    t = c;
    c = d;
    d = t;
}
```

* executing the program in a compiler having call by copy-restore by default.



5/3*5
3/5*5

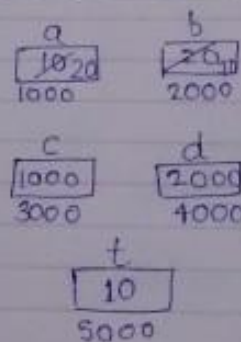
* Changes to actual parameter takes place at the end using call by copy restore.

* Changes to actual parameter takes place on the fly using call by reference.

Call by reference

```
main()
{
    int a=10, b=20;
    printf("%d %d", a, b);
    swap(a,b);
    printf("%d %d", a, b);
}

swap(int *c, int *d)
{
    int t;
    t = *c;
    *c = *d;
    *d = t;
}
```



o/p:-
10 20 | 20 10

Q. Consider the following program:-

```
main()
{
    int a = 5;
    Test(a, a);
    printf(a);
}
Test(int c, int d)
{
    c += d * 2;
    d *= c - 3;
}
```

A call by reference or call by copy restore gives diff. answer if the program contains aliasing.

★ If the program contains aliasing then call by reference & call by copy restore may generate same result.

Call by Value:-

a	c	d
5	15	60
1000	2000	3000

$c += d * 2 \rightarrow c = c + d * 2$

$$= 5 + 10$$

$$= 15$$

$d *= c - 3 \rightarrow d = d * (c - 3)$

$$= 15 * (12)$$

$$= 180$$

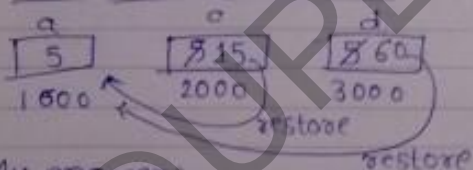
$$= 60$$

in this case, always R.H.S. is calculated first irrespective of the priorities.

O/p:-
5

printf(a)

Call by Copy-Restore:-



My answer:-

O/p:- 60 15 60

whichever is restored last is the output.

Call by reference:-

a	c	d
5	1000	1000
1000	2000	3000

$*c += (*d) * 2;$

$(*c) = (*c) + ((*d) * 2);$

$$(*c) = 5 + (5 * 2)$$

$$= 15$$

$(*d) = (*d) * ((*c) - 3);$

$$= 15 * (15 - 3)$$

$$= 15 * 12$$

$$= 180$$

O/p:-
180

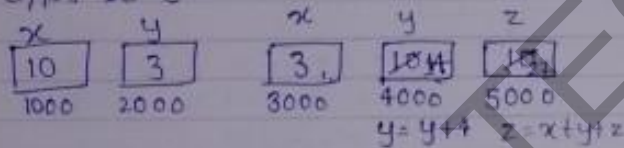
Q. Consider the following programs:-

```
main()
{
    int x=10, y=3;
    fun(y, x, x);
    printf("%d", y);
}

fun(int x, int y, int z)
{
    y = y+4;
    z = x+y+z;
}
```

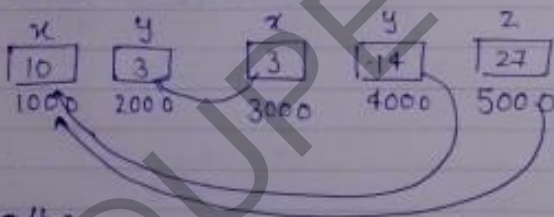
Call by Value:-

O/p:- 10 3



Call by Copy restore:-

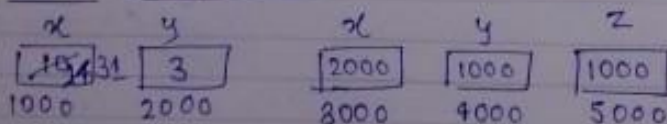
same as above



O/p:-

14/27 3

Call by reference



$$\begin{aligned} *y &= *y + 4 \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

$$\begin{aligned} *z &= (*x) + (*y) + (*z) \\ &= 10 + 14 + 14 \\ *z &= 38 \end{aligned}$$

O/p:-

31 3

Aliasing :- 2/4

• int x; $\rightarrow$ 1000

int &y=x; $\rightarrow$ means y address is same as x address.
 *C doesn't support aliasing.

Q. Imp

main()

{ int a,b;

a=2; b=3;

P(a+b, a);

printf(a,b);

}

P(int x, int y, int z)

{ y=y+1;

z=z+x;

x=x+y+z;

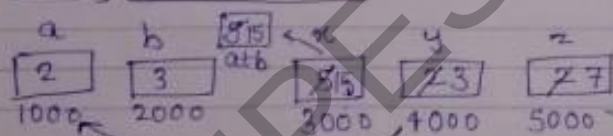
}

this is done
in temporary
variable & its
address is passed.

Call by Value:-

O/p:- 2, 3

Call by Copy restore:-



a=3/7

a+b=15

if a=3, then b=12

if a=7, then b=8

My answer

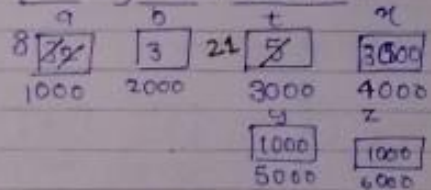
$\therefore$ O/p:-

3/7 12/8 $\rightarrow$ My answer

but answer:-

3/7 3

Call by reference:-



*y = *y+1

= 2+1

= 3

*z = (*z) + (*x)

= 3+5=8

*x = (*x) + (*y) + (*z)

= 5+8+8

= 21

O/p:-

8 3

```

Q. main()
{
    int a=10, b=20;
    swap(a,b);
    printf(a,b);
}

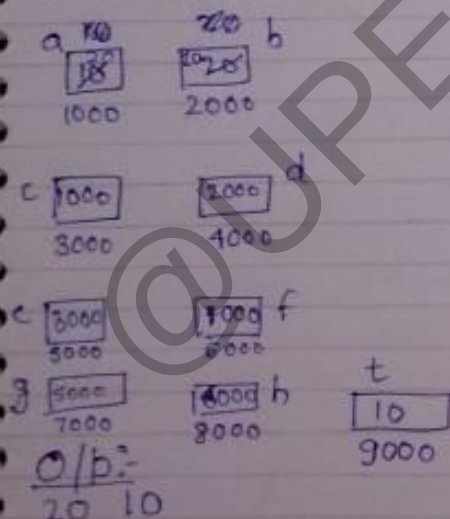
swap(int*c, int*d)
{
    interchange(&c, &d);
    printf(c,d);
}

interchange(int*e, int*f)
{
    A(&e, &f);
}

A(int**g, int**h)
{
    int t;
    t=*g; // t=***g;
    *g=*h; // ***g=***h;
    *h=t; // ***h=t;
}

```

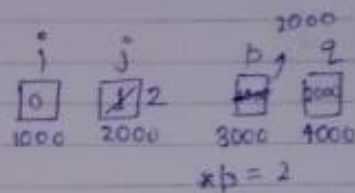
Call by reference:-



Q. Consider following C-program:-

```
#include <stdio.h>
void f(int *p, int *q)
{
    *p = 2;
}
int i = 0, j = 1;
main()
{
    f(&i, &j);
    printf("%d %d", i, j);
    return(0);
}
```

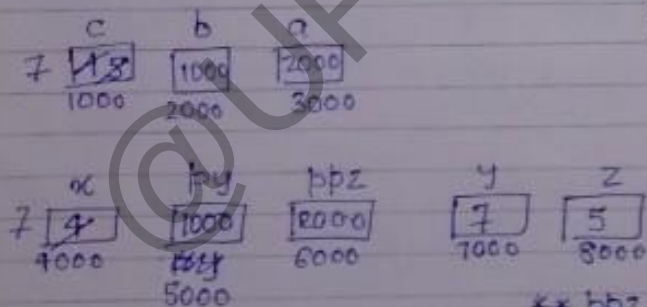
Output:- 0 2



Q. What is the o/p printed by the following C-program:-

```
main()
{
    int c, *b, **a;
    c = 4;
    b = &c;
    a = &b;
    printf("%d", f(c, b, a));
}

f(int x, int *py, int **ppz)
{
    int y, z;
    **ppz += 1;
    z = **ppz;
    *py = 2;
    y = *py;
    x += 3;
    return(x + y + z);
}
```



$$7 + 7 + 5 = 19$$

O/p:-

19

```
**ppz = **ppz + 1;    y = *py;
                     = 2000 + 1 = 2001    y = 7
z = **ppz
  = 2001
*py = *py + 2
   = 1000 + 2 = 1002
x = x + 3
  = 4 + 3 = 7
```

Q. Consider the following C program:- (Static Scoping, by default as its a C program)

```
#include <stdio.h>
int x;
void Q(int, z)
{
    z += x;
    printf(z);
}
```

```
void P(int *y)
{
    int x = *y + 2;
    Q(x);
    *y = x - 1;
    printf(x);
}
```

```
main()
{
    x = 5;
    P(&x);
    printf(x);
}
```

x
6 [8]
1000

y
[1000]
2000

x (of P)
[7]
3000

$x = *y + 2$
 $= 5 + 2$
 $= 7$
 $*y = 7 - 1 = 6$

z
[7] 12
4000
 $z += x$
 $= 7 + 5 (\text{global})$
 $= 12$

O/p:-

[12, 7, 6]

Q. int a[5] = {10, 20, 30, 40, 50};

a →

1000	1002	1004	1006	1008
10	20	30	40	50

 1000 to 1008

• array name indicates starting element address
• variable name indicate inside value.

a = 1000

*a = 10

a + 1 = 1000 + 1 * 2 = 1002 [as integer is of 2 bytes]

*(a + 1) = *(1002) = 20 = a[1]

*1 + a

*(a + 4) = 50 = a[4]

★ at time of preprocessing a[4] is converted to *(a + 4).

a[50] = *(a + 50) = *(base address + 50 * 2)

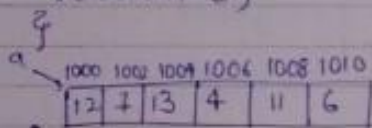
↑ random access is possible due to pointers.

- a + 0 → skipping 0 elements → a[0]
- a + 1 → " 1 " → a[1]
- a + 2 → " 2 " → a[2]


```

main()
{
    int a[] = {12, 7, 13, 4, 11, 6};
    printf("%d", f(a, 6));
    return 0;
}

```



15

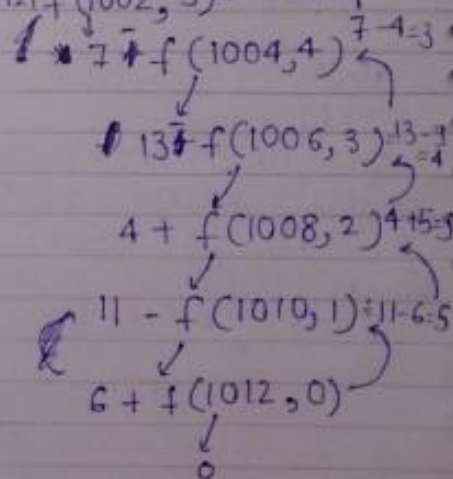
$\alpha(f)$	m
1000	6

2000 3000

$$*a \frac{1}{2} = 12 \frac{1}{2} = 0$$

$$x_a + f(a+1, n-1) = 12 + 3 = 15$$

$$\begin{aligned}
 & 12 + (7 - (13 - (4 + (11 - (6 + 0)))))) \\
 &= 12 + (7 - (13 - (4 + 5))) \\
 &= 12 + (7 - (13 - 9)) \\
 &= 12 + (7 - 4) \\
 &= 12 + 3 = 15
 \end{aligned}$$



Q. What is the O/p of the following C-program?

```
#include <stdio.h>
int f(int n, int k)
{ if (n == 0)
  return 0;
  else
    if (n % 2)
      return f(n/2, 2*k) + k;
    else
      return f(n/2, 2*k) - k;
}
```

```
main()
{ printf("%d", f(20, 1));
}
```

O/P :-
9

Diagram illustrating the recursive calls for $f(20, 1)$:

```

f(20, 1)
├── f(10, 2) - 1 = 10 - 1 = 9
│   ├── f(5, 4) - 2 = 12 - 2 = 10
│   │   ├── f(2, 8) + 4 = 8 + 4 = 12
│   │   │   ├── f(1, 16) - 8 = 16 - 8 = 8
│   │   │   │   ├── f(0, 32) + 16 = 16
│   │   │   │   │   └── 0

```

Q. Main()

```
{ int a[] = {10, 20, 30, 40, 50};
  int *p[] = {a, a+1, a+2, a+3, a+4};
  int **ptr = p;
  ptr++;
```

```
printf("%d %d %d", ptr-p, *ptr-a, **ptr);
      *(++ptr);
```

```
printf("%d %d %d", ptr-p, *ptr-a, **ptr);
      ++(*ptr);
```

```
printf("%d %d %d", ptr-p, *ptr-a, **ptr);
      *ptr++;
```

```
printf("%d %d %d", ptr-p, *ptr-a, **ptr);
}
```

ptr++ = ptr
& after semicolon
is encountered
then only ptr
is incremented

a

10	20	30	40	50
1000	1002	1004	1006	1008

p

1000	1002	1004	1006	1008
2000	2002	2004	2006	2008

O/p:

2	2	20
4	4	30
4	6	40

ptr

2000
3000

→ ptr++, → ptr = ptr + 1 →

2002
3000

(it gives no. of elements)

1st point f → ptr - p = 2002 - 2000 = 2 → 2 means only 1 element

*ptr - a = 1002 - 1000 = 2 → 2 means only 1 element

**ptr = 20

→ *(++ptr) →

2004
3000

2nd point f → ptr - p = 2004 - 2000 = 4 → 4 means '2' no. of elements

*ptr - a = 1004 - 1000 = 4 → 4 means '2' no. of elements

**ptr = 30

→ ++(*ptr) → *ptr = 1004, ++(1004) = 1006

(*ptr) = (*ptr) + 1;

ptr

2000	2002	2004	2006	2008
1000	1002	1004	1006	1008

ptr

2004
3000

→ 3rd point f → ptr - p = 2004 - 2000 = 4 → 4 means '2' no. of elements

*ptr - a = 1006 - 1000 = 6 → 6 means '3' no. of elements

**ptr = 40

→ *ptr++; → *ptr will take first → 1006

ptr will not take place till is encountered
*ptr takes place 1st, & after execution ptr is incremented

p

1000	1002	1004	1006	1008
------	------	------	------	------

ptr

2006
3000

(*ptr) = (*ptr) + 1;
after this statement, ptr will take place

★ Note:- '*' and '++' having same priority, so we will go to associativity, & associativity is right to left.

→ 4th printf statement → $ptr - p = 2006 - 2000 = 6$ → 6 means 3 no. of elements
 $*ptr - a = 1006 - 1000 = 6$ → 6 means 4 no. of elements
 $**ptr = 5040$

★ If two addresses undergoes arithmetic opn.

O/p

1 1 20
 2 2 30
 2 3 40
 2 4 50 3 3 40

Q. #include <stdio.h>

main()

{ int a[] = {0, 1, 2, 3, 4};

int *p[] = {a, a+1, a+2, a+3, a+4};

int **ptr = p;

**ptr++;

printf("/d/d/d", ptr-p, *ptr-a, **ptr);

*++*ptr;

printf("/d/d/d", ptr-p, *ptr-a, **ptr);

++**ptr;

printf("/d/d/d", ptr-p, *ptr-a, **ptr);

}

a

0	1	2	3	4
1000	1002	1004	1006	1008

b

1000	1002	1004	1006	1008
2000	2002	2004	2006	2008

ptr

2000

3000

→ **ptr++;

↳ from right to left

ptr++ will take place 1st

∴ ptr++ ⇒ ptr = 2000

& *ptr = 1000

**ptr = 0

& after ; , ptr++ , ∴ ptr = 2002

O/p:-

1	1	1
1	2	2
1	2	3

1st printf:- $ptr - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*ptr - a = 1002 - 1000 = 2 \rightarrow$ " " " "
 $**ptr = 1$

$\rightarrow *++*ptr \rightarrow$ from right to left
 $*ptr \rightarrow 1002$
 $++*ptr \rightarrow (*ptr) = (*ptr) + 1$

$\therefore p \rightarrow$

1000	1004	1004	1006	1008
2000	2002	2004	2006	2008

$*1004 = 2$

2nd printf:- $ptr - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*ptr - a = 1004 - 1000 = 4 \rightarrow 4$ " '2' elements
 $**ptr = 2$

$\rightarrow ++**ptr \rightarrow$ from right to left
 $*ptr = 1004$
 $**ptr = 2$
 $++**ptr \rightarrow **ptr = **ptr + 1$

$\therefore a$

1000	1002	1004	1006	1008
0	1	3	3	4

$\therefore$ 3rd printf:- $ptr - p = 2002 - 2000 = 2 \rightarrow 2$ means '1' element
 $*ptr - a = 1004 - 1000 = 4 \rightarrow 4$ " '2' element
 $**ptr = 3$ [$*ptr = 1004 \rightarrow **ptr = 3$]

★ Subtracting two pointers means $*(**ptr)$ gives the total size of the elements $*(**ptr++)$ that can reside in b/w them.
 if the elements are integers, then
 total no. of int = total size b/w them

Note 1:- For the pointer variable, we can add integer constants.
e.g. $p = p + 5$ $p_1 + i$ (skipping i elements in forward direction).

Note 2:- We can subtract integer constant from pointer variable
e.g. $p_1 = 1014$
 $p_1 = p_1 - 1 = 1014 - 2 \times 1 = 1012$
 $p_1 - i$ (skipping i elements in backward direction)
e.g. $p = 1000$ $q = p$ given no. of elements in
 $q = 1008$

Note 3:- Subtraction b/w 2 pointers is possible (p, q) iff
 $p > q$ [$p \geq q$ should be pointers of the same array]
where ' $p - q$ ' indicates no. of elements b/w two pointers including 1st one & excluding last one.

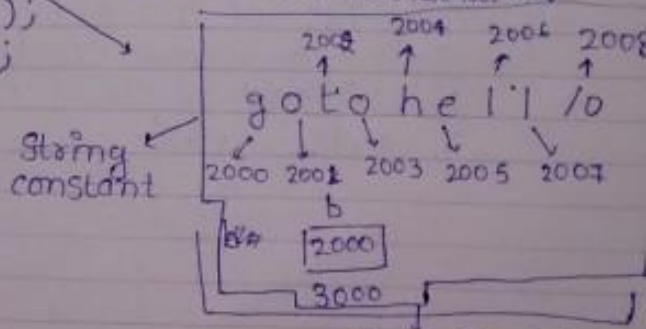
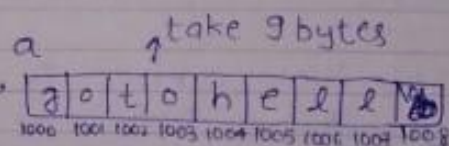
Note 4:- Multiplication, division & addition is not possible b/w two pointers because it has no meaning.

Q. main() Array of characters

```

char a[] = "gotohell";
char *b = "gotohell";
a[3] = printf("%s", a);
printf("%s", b);
a[3] = 'w';
b[3] = 'w';
printf("%s", a);
printf("%s", b);

```



* $a = a + 5$ → error
(we can't change the base address of the array.)

* but $b = b + 3$ → no error
because b is character pointer & can be changed.

take $8 + 2$ bytes
Char pointer
= 11 Bytes

* `printf("%s", 1000)` → means it will print characters starting from 1000 address till null character (null character is not printed.)

* `printf("%s", *1000)` → it will print only one character at 1000 address.

• `printf("%s", a)` → goto hell
• `printf("%s", a+4)` → hell
• `printf("%s", *(a+1))` → l
• `printf("%s", b)` → goto hell
• `printf("%s", b+5)` → cll
• `printf("%s", *(b+5))` → e
• `printf("%s", *(a+3))` → 0

• `a[3] = *(a+3)`

`* (a+3) = 'w'`

• `a`

j	o	t	w	h	e	l	l	\0
---	---	---	---	---	---	---	---	----

• `b[3] = *(b+3)`

`printf("%s", a);` → goto hell

`printf("%s", b);` → goto hell

Note 1:- Base address of the array can't be changed by string constant can be changed.
Contents of the array can be changed, but string constant content can't be changed, if contents of string constant are changed, result is undefined.

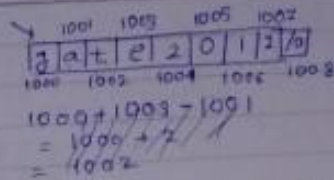
Date

05.08.12

```
main()
{ char p[] = "gate2012";
  printf("%s", p+p[3]-p[1]);
}
```

O/P:

te2012 2012



$$p[3] = *(p+3) = \text{'e'}$$

$$e - a = 1004 - 1001 = 3$$

$$p[1] = *(p+1) = \text{'a'}$$

Warning in compilation takes place (program)
When there is no loss of data but it doesn't follow semantic rules.

eg. int a;

char b;

a = b; // only warning, implicitly it is converting a = (int) b
// coercion (implicit type conversion)

// b = a; → this generate error message, because loss of data
// we are storing 2 byte data into 1 byte storage, & hence error.

b = (char) a; // correction, explicit conversion.

★ printf("%c", 'a'); → 'a'

★ printf("%d", 'a'); → 97

★ Lower Address → Higher Byte } → Big endian
Higher " → Lower Byte }

★ Lower Address → Lower Byte } → Little endian
Higher " → Higher " }

★ char s = 'a';

s = s + 1; // s now contains 'b'

★ char s = 255;

s = s + 1; // s will be 0.

★ char s;

s = 'a' + 'b'; → s = 97 + 98 = 195

s = 'b' - 'a'; → s = 98 - 97 = 1

Semantic rule

Lower byte variable using lower memory can be stored in a variable using more memory but not vice versa.

★ $* (p+i) = p[i]$
 ★ $* (i+p) = i[p]$ $\rightarrow$ These two are same.

★ `printf ("%s", p);` $\rightarrow$ gate2012

★ `printf ("%s", p+5);` $\rightarrow$ 012

★ `printf ("%c", *(p+5));` $\rightarrow$ 0

★ `printf ("%c", p);` $\rightarrow$ error message, because we are using address with %c

★ `printf ("%s", *p);` $\rightarrow$ error message, as *p means j.

★ `printf ("%c", *p);` $\rightarrow$ j

Q. `main()`

```
{ char p[20];
```

```
  char *s = "string"; // string constant
```

```
  int length = strlen(s);
```

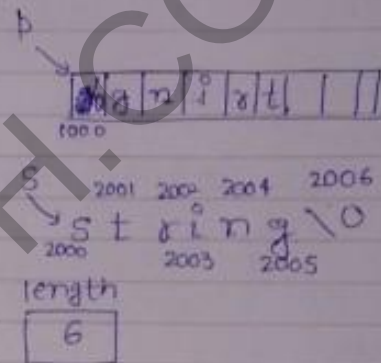
```
  for (int i = 0; i < length; i++)
```

```
  { p[i] = s[length-i];
```

```
  }
```

```
  printf ("%s", p);
```

O/p :- Nothing (because first character is '\0' itself).



★ `sizeof("abcd");` $\rightarrow$ 5 (including /0) sizeof means the size of "abcd" in the memory, i.e. 5 Byte.

★ `strlen("abcd");` $\rightarrow$ 4

Q. `main()`

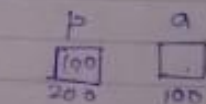
```
{ char *p;
```

```
  printf ("%d %d", sizeof(p), sizeof(*p));
```

```
}
```

O/p :- 4 1 $\rightarrow$ *p means p is pointing to a character, which takes 1 B.

4 1 $\rightarrow$ sizeof(p) means p store address, address takes 4 B.



Q. `main()`

```
{ char *p = "good";
```

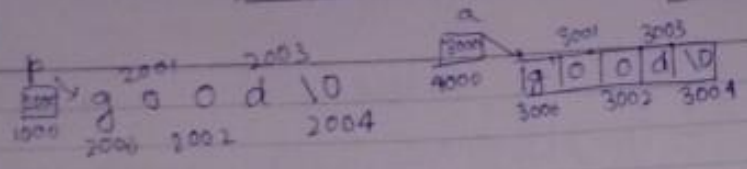
```
  char a[] = "good";
```

```
  printf ("%d %d %d", sizeof(p), sizeof(*p), strlen(p));
```

```
  printf ("%d %d", sizeof(a), strlen(a));
```

```
}
```

Note:- void a=10; → sizeof(a) → error
 void *b=4a; → sizeof(b) → b is a pointer which holds an address & hence takes 4B.



O/P:-
 4 1 3 4
 4 3 5 4 3 4
 When we take address to be taking 4 Bytes
 sizeof(*p) → p is pointing to a character, which is of 1B.
 sizeof(p) → p stores an address

void a;
 int b;
 float c;
 char d;
 a=b;
 a=c;
 a=d;

★ Any datatype can be stored in void variable & void can be stored in any variable (with loss of data).
 (without loss of data)
 b=(int)a; // have to do explicit typecasting
 c=(float)a;
 d=(char)a;

→ no error, without typecasting this is possible
 advantage of void type

if:- int a[5];
 sizeof(a); → size of the array :- 10B
 char a[] = "good";
 sizeof(a); → size of the array :- 5B (including null character)

```
main()
{
  int i=-1, j=-1, k=0, l=2, m;
  m = ((i++ && j++ && k++) || l++); m=1
  printf("%i %i %i %i %i", i, j, k, l, m);
}
```

O/P:-
 0 0 1
 My answer:- -1 -1 1 2 1
 But answer:-
 → this whole statement is true, no need to do l++
 m = ((i++ && j++ && k++) || l++);
 left to right because && is binary operator
 -1 && -1 && -1 = true = 1
 O/P:- 0 0 0 2 1, l++ wasn't done.

★ if (0:10)
 printf("hi");
 else
 printf("bye");
 O/P:- hi
 ★ else A=10
 B=0
 if (A=B)
 printf("hi");
 else
 printf("bye");
 O/P:- bye

★ if $k=0$

then o/p - 0 0 10 3 1

is false

(i++ && j++ && k++)

we need to execute i++

★ If $i=0, j=2, k=3$ & $l=2$

∴ o/p :- 1 2 3 3 1

Imp.

★ `int a[10] = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100}`

10001001									
10	20	30	40	50	60	70	80	90	100
0	1	2	3	4	5	6	7	8	9

`int *b, = (int *) a;`

`char *c;` // c waiting for an address of character.

`float *d;`

`void *e;`

`c = (char *) a;` →

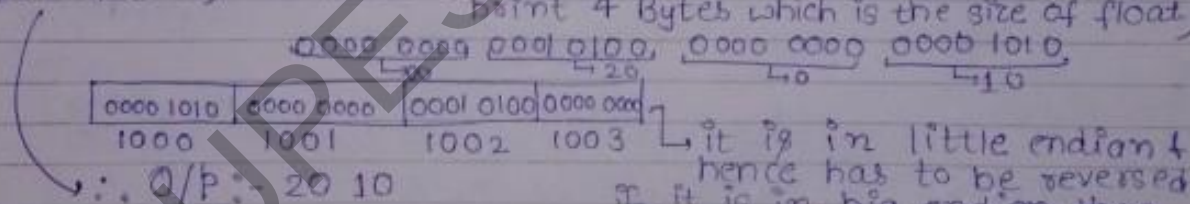
1000
3000

`d = (float *) a;` →

1000
4000

`printf(d)` → 1000

`printf(*d)` → 10 20 (because d is a float pointer & will print 4 Bytes which is the size of float)



`printf(c)` → 1000

`printf(*c)` → only 1 Byte will be retrieved, ∴ only 1000 address value is printed, ∴ o/p - 10

`printf(e)` → 1000

`printf(*e)` → error.

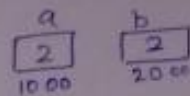
★ if `c++`, then `c = 1001`

★ if `d++`, then `d = 1004`

★ if `b++`, then `b = 1002`

★ if `e++`, then error.

* No compilation errors,
but windows stopped
the program execution.



Q.

```
int a=2;
int *b=(int *)a; → 

|      |
|------|
| 2    |
| 2000 |

 → 2 is an integer pointer now, which is integer value before
*if(c) printf(b) → 2
printf(*b) → garbage (value at address 2)
++b → 4 (because 2 is now integer pointer & hence ++b will increment it to 4)
++a → 3 (because value in a is integer value.)
```

Q. int a=2;

```
float *b=(float *)a; → 

|      |
|------|
| 2    |
| 2000 |

 → integer value '2' is converted to floating pointer
printf(b) → 2
* printf(*b) → garbage (value at address 2, 3, 4, 5)
++b → 6 (because floating pointer increment it by 4)
++a → 3
```

Q. int a=2;

```
char *b=(char *)a; → 

|      |
|------|
| 2    |
| 2000 |

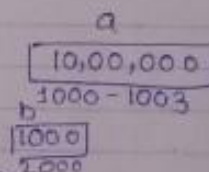
 → integer value '2' is converted to character pointer
printf(b) → 2
* printf(*b) → garbage (value at address 2)
++b → 3 (increment by 1 only because character pointer)
++a → 3
```

Q. float a = 10,00,000; → takes 4 Bytes

```
int *b = (int *)&a;
```

```
printf(b) → 1000
```

```
* printf(*b) → since b is an integer pointer, hence will print 2 Bytes out of 4 Bytes (hence data loss)
```



★ If address lines are of 2 Bytes

then float a = 10,00,000;

```
int *b = (int *)&a
```

we are storing a (4 Bytes data) into 2 Bytes integer pointer & hence the 4 Byte a (converted to integer pointer) is stored in 2 Byte 'b' & hence address loss.

Static Variable

main()

```
{ static int var = 5;
  printf("%d", var);
  if (var)
    main();
}
```

O/p:-

54321

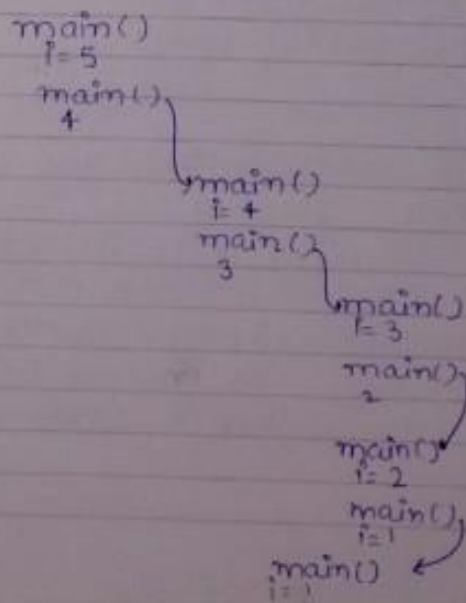
★ Writing 'static' means at the time of compilation time memory is allotted. (Static binding) [compile time memory is allocated]

★ if 'static' isn't there then the memory is allotted at run time. (dynamic binding) [memory is allocated at run time]

- ① For the static variable memory will be created only once during its life time (compile time only).
(Re Compiling the program will allocate different memory location to the static variable.)
- ② For the static variable initialization will be done once.
(By default)
- ③ By default static & global variable are initialized to zero.

Q. void main()
{ static int i = 5;
 if (--i)
 { main();
 printf("%d", i);
 }
}

O/p:-
0000



```

Q) main() { int f(int n)
    { static int x=0;
      if(n<=0)
        return 1;
      if(n>3)
        { x=n;
          return f(n-2)+2;
        }
      return f(n-1)+x;
    }
}

```

What is the value of $f(5)$?

$f(5)$
 $(n>3)$
 $\rightarrow$ return $f(n-3)+2 = 16+2 = 18$
 $f(3)$
 $\rightarrow$ return $f(2)+x = 11+5 = 16$
 $f(2)$
 $\rightarrow$ return $f(1)+x = 6+5 = 11$
 $f(1)$
 $\rightarrow$ return $f(0)+x = 6$
 $f(0)$
 $\rightarrow$ return 1

x
 $\boxed{5}$
 1000

o/p:-
 18

Note:- If we use register
 instead of static,
 then x is stored
 in register for
 faster access.

```

#include <stdio.h>
main()
{ struct a
  { char c[7];
    char *str;
  };
  struct b

```



```

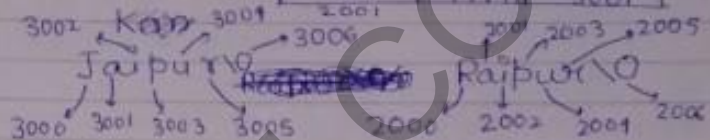
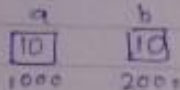
{ char *c;
  struct a s1;
};
struct b s2 = {"Raipur", "Kampur", "Jaipur"};
printf("%s/%s", s2.c, s2.s1.st);
printf("%s/%s", ++s2.c, ++s2.s1.st);
}

```

O/p:-

Raipur Jaipur
aipur aipur

★ int a=10;
int *b=a; //no error



Structures & Union

```

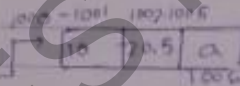
struct s1
{
  int a;
  float b;
  char c;
};

```

```

struct s1 x;
x = {10, 20.5, 'a'};

```



NOTE:- Whenever we reference a struct, many variables are referenced, hence selection is req. to select the intended variable using $\dots$

- ★ printf(x) → //means nothing first element of the struct will be printed.
- ★ printf(x.a) → 10
- ★ printf(x.b) → 20.5
- ★ printf(x.c) → a
- ★ if struct s1

if int a;
then s1.a not required s1 will also print a.
Both are possible

```

struct s1
{
  int a;
  float b;
  char c;
};

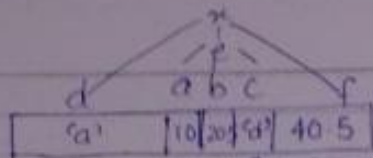
```

```

struct s2
{
  char d;
  struct s1 e;
  float f;
};

```

we can declare struct within struct.



struct s = { 'a', 10, 20.5, 'd', 40.5 }

struct x = { 'a', 10, 20.5, 'd', 40.5 }

x.d = 'a'

x.e.a = 10

x.e.b = 20.5

x.e.c = 'd'

x.f = 40.5

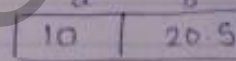
Q struct s

{ int a;

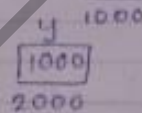
float b;

};

1 struct s, x = { 10, 20.5 };



2. struct s, *y = &x;



3. printf(y) → 1000

printf((*(y)).a) → 10

or printf(y->a); → 10

Q. main()

{ struct s

{ char *z;

int i;

struct s, *p;

};

struct s, a[] = { { "Nagpur", 1, a+1 },

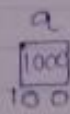
{ "Raipur", 2, a+2 },

{ "Kamapur", 3, a+3 } };

struct s, *ptr = a;

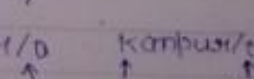
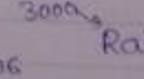
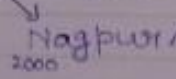
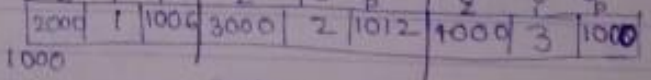
printf("%s/%s/%s/%s/%s", a[0].z, ptr->z, a[2].b->z);

(*ptr).z a[2].(*ptr).z



O/P:-

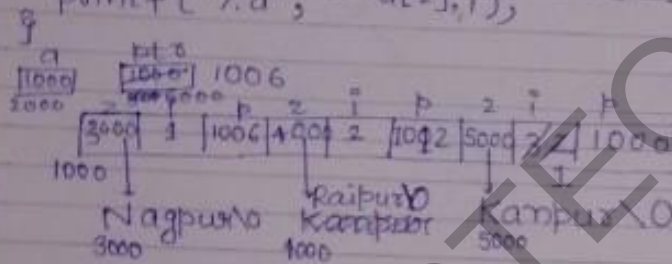
Nagpur Nagpur Nagpur



★ processor will change $p \rightarrow c$ into $(*p).c$

```
main()
{
    struct s1
    {
        char *z;
        int i;
        struct s1 *p;
    };
    struct s1 a[] = {{"Nagpur", 1, a+1}, {"Raipur", 2, a+2}, {"Kampur", 3, a+3}};
    struct s1 *pt = a;
    printf("%s", ++(pt->z));
    printf("%s", a[(pt->z)->i].z);
    printf("%s", a[--(pt->z)->i].z);
    printf("%d", --a[2].i);
}
```

O/p:-
agpur
Kampur
Kampur
21



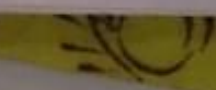
```
Q. #include <stdio.h>
struct Test
{
    int i;
    char *c;
};
struct Test st[] = {5, "become", 4, "better", 6, "jungle", 8, "ancestor", 7, "brother"};
```

```
main()
{
    struct Test *p = st;
    p = p+1;
    printf("%s", ++(p->c));
    printf("%c", *++(p->c));
    printf("%d", p[0].i);
    printf("%s", p->c);
}
```

$(*p).c++$

O/p:-
etter, u, e, ungle

$p[0] = *(p+0)$



```

Q. void main()
{
    char s[] = "hello";
    char R[] = "hello";
    if (S == R)
        printf("equal");
    else
        printf("not equal");
}

```

★ if *S == *R ('h' == 'h')
O/p:- equal

O/p:-

Not equal

because S & R both stores
diff. base addresses.

★ Arrays

1-D (1-Dimensional)

```
int a[10] = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100};
```

1000	1002	1004	1006	1008	1010	1012	1014	1016	1018
10	20	30	40	50	60	70	80	90	100
0	1	2	3	4	5	6	7	8	9

$$a[5] = 1000 + 2 \times 5 = 1010$$

$$L(a[5]) = 1000 + 2 \times (5 - 0) = 1010$$

$$L(a[9]) = 1000 + 2 \times (9 - 0) = 1018$$

if array starts with index '1'

$$L(a[5]) = 1000 + 2 \times (5 - 1) = 1008$$

$$L(a[9]) = 1000 + 2 \times (9 - 1) = 1016$$

in C compiler it is like
 $1000 + 2 \times 5$
(So C compiler saves '1' subtraction.)

This means before
5th element, how many
elements are there.

Q. Array A is defined as follows:-

- A[-5...+5]
- Base address (BA) = 999
- Size of element = 100B

Location of a[-1], a[+5]

$$L(a[-1]) = 999 + 100 \times (-1 - (-5)) = 999 + 100 \times 4 = 1399$$

$$L(a[5]) = 999 + 100 \times (5 - (-5)) = 999 + 100 \times 10 = 1999$$

Total no. of elements:-

$$\boxed{\text{Last Index} - \text{1st Index} + 1}$$

In General :-

- $A[lb \dots ub]$
 - $BA \rightarrow$ Base Address
 - $C \rightarrow$ Size of 1 element
- then location of i^{th} element -
- $$Loc(a[i]) = B.A + C \times (i - lb)$$

Two-Dimensional Arrays

int a[4][4];

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

★ Every multidimensional array is stored in the form of multidimensional or 1-dimensional array.

Row-Major Order :-

Q

1000	1002	1004	1006	1008	1010	1012	1014	1016	1018	1020	1022	1024	1026	1028	1030
a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}	a_{24}	a_{31}	a_{32}	a_{33}	a_{34}	a_{41}	a_{42}	a_{43}	a_{44}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

$$L(a[i][j]) = B.A + C \times ((i - lb_1) \times (ub_1 - lb_1) + (j - lb_2) \times (ub_2 - lb_2))$$

$$L(a[1][3]) = 1000 + (4(4-1) + (3-1)) \times 2$$

$$= 1000 + 28$$

$$= 1028$$

$$Loc(a[2][4]) = 1000 + (4(2-1) + (4-1)) \times 2$$

$$= 1000 + 7 \times 2$$

$$= 1014$$

Total Rows

Last Row No. -
1st Row No. + 1

Total Columns

Last Column No. -
1st Column No. + 1

array starting from $a[1][1]$

Q Consider the following 2-D array :-

$a[5, 25, 50, 75]$

$B.A = 0$

$C = 5B$

$$\begin{aligned} L(a[20][70]) &= 0 + (70-50)(20-5) \times (75-50) + (70-50) \times 5 \\ &= (15 \times 25 + 20) \times 5 \\ &= 395 \times 5 = 1975 \\ &= 410 \times 5 = 2050 \end{aligned}$$

$$\begin{aligned} Loc(a[25][75]) &= 0 + ((25-5) \times (75-50+1) + (75-50)) \times 5 \\ &= (20 \times 26 + 25) \times 5 \\ &= 445 \times 5 = 2225 \\ &= 545 \times 5 = 2725 \end{aligned}$$

In General (Row Major Order)

$A[lb_1, \dots, ub_1, lb_2, \dots, ub_2]$
 BA, C

$$Loc(a[i][j]) = BA + ((i - lb_1) \times (ub_2 - lb_2 + 1) + (j - lb_2)) \times C$$

Date _____

11.08.12

```
• int a[4][4];
```

$$a_{11} \quad a_{12} \quad a_{13} \quad a_{14}$$
$$a_{21} \quad a_{22} \quad a_{23} \quad a_{24}$$
$$a_{31} \quad a_{32} \quad a_{33} \quad a_{34}$$
$$a \quad a_{41} \quad a_{42} \quad a_{43} \quad a_{44}$$
$$a[lb_1 \quad ub_1][lb_2 \quad ub_2]$$

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30
a_{11}	a_{21}	a_{31}	a_{41}	a_{12}	a_{22}	a_{32}	a_{42}	a_{13}	a_{23}	a_{33}	a_{43}	a_{14}	a_{24}	a_{34}	a_{44}

$$\text{Loc}(a[i][j]) = B.A. + ((j - lb_2) \times (ub_1 - lb_1 + 1) + (i - lb_1)) \times C$$

$$\begin{aligned} \cdot \text{Loc}(a[3][+]) &= 1000 + ((4-1) \times 4 + (3-1)) \times 2 \\ &= 1000 + (12 + 2) \times 2 \\ &= 1000 + 28 \\ &= 1028 \end{aligned}$$

$$\begin{aligned} \cdot \text{LOC}(a[4][3]) &= 1000 + ((3-1) \times 4 + (4-1)) \times 2 \\ &= 1000 + (8+3) \times 2 \\ &= 1000 + 22 \\ &= 1022 \end{aligned}$$

$$\begin{aligned} \cdot \text{Loc}[a[4][4]) &= 1000 + ((1-1) \times 4 + (4-1)) \times 2 \\ &= 1000 + 6 = 1006 \end{aligned}$$

Q. Consider the array (using Column Major Order)

$$A[-5 : +50][25 : 75]$$

- BA-999

- C - 10 B

$$\begin{aligned} \text{Loc}(A[0][74]) &= 999 + ((74-25) \times (20-(5)+1) + (0-(5))) \times 10 \\ &= 999 + (49 \times 26 + 5) \times 10 \\ &= 999 + 12790 \\ &= 13789 \end{aligned}$$

$$\begin{aligned}\text{Loc}(A[-2][30]) &= 999 + ((30-25) \times 26 + (-2+5)) \times 10 \\ &= 999 + (130+3) \times 10 \\ &= 999 + 1330 = 2329\end{aligned}$$

$$\begin{array}{r} 5 \\ 49 \\ \times 26 \\ \hline 294 \\ 980 \\ \hline 1274 \\ + 3 \end{array}$$

Lower Triangular Matrix

int a[4][4]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

*it is valid only for square matrices.

$$A[i][j] = 0 \text{ if } i < j$$

Storing using Row Major :-

a	1000	02	04	06	08	10	12	14	16	18
	a ₁₁	a ₂₁	a ₃₁	a ₄₁	a ₁₂	a ₂₂	a ₃₂	a ₄₂	a ₁₃	a ₂₃
	1st	2nd	3rd	4th						

$$\begin{aligned} \text{LOC}(a[4][3]) &= 1000 + ((4-1) \times ((1+2+3) \times (4-1) + (3-1))) \times 4 \\ &= 1000 + 18 \\ &= 1000 + ((4-1)(1+2+3) + (3-1)) \times 2 \\ &= 1000 + (8) \times 2 \\ &= 1016 \end{aligned}$$

$$\text{LOC}(a[i][j]) = B.A + ((i-lb_i)NNS + (j-lb_j)) \times C$$

$(i-lb_i)$ → Natural No Sums upto from 1 to $(i-lb_i)$

$(4-1)$ sum of natural nos. (i.e. sum of natural nos. upto 3.)

$$\begin{aligned} \text{LOC}(a[4][3]) &= 1000 + ((1+2+3) + (2-1)) \times 2 \\ &= 1000 + (6+1) \times 2 = 1014 \end{aligned}$$

Q. Consider matrix defined as follows:-

A[-25...+25, -25...+25]

B.A = 0

Row Major Order, lower Triangular Matrix

C = 100 B

$$\begin{aligned} \text{LOC}(A[-20][-21]) &= 0 + ((-20+25)NNS + (-21+25)) \times 100 \\ &= 0 + ((1+2+3+4+5) + 4) \times 100 \\ &= 0 + (19) \times 100 \\ &= 1900 \end{aligned}$$

$$\frac{5 \times 4 \times 3}{2} = 1$$

$$\begin{aligned} \text{Loc}(A[0][0]) &= 0 + ((0+25) \text{NNS} + (0+25)) \times 100 \\ &= 0 + \left(\frac{25 \times 26^{13} + 25}{2} \right) \times 100 = 35,000 \end{aligned}$$

Column Major Order:-

00	02	04	06	08	10	12	14	16	18
a ₁₁	a ₂₁	a ₃₁	a ₄₁	a ₂₂	a ₃₂	a ₄₂	a ₃₃	a ₄₃	a ₄₄

My answer

$$\text{Loc}(a[i][j]) = \text{B.A} + \left(\left(\frac{10 \times 10}{2} \right) \text{reverse order sum} + \left(\frac{9-j}{2} \right) \right) \times C$$

$$\begin{aligned} \text{Loc}(a[4][3]) &= 1000 + \left(\left(\frac{4 \times 5}{2} \right) + (4-3) \right) \times 2 \\ &= 1000 + (7 + 1) \times 2 \\ &= 1010 \quad 1020 \quad 1016 \end{aligned}$$

★

or $\text{Loc}(a[4][3]) = 1000 + \left(\left(\frac{4 \times 5}{2} \right) - (4-3+1) \text{NNS} + (4-3) \right) \times 2$

skip all columns → we actually want to go to 3rd column

a ₁₁	a ₁₂	0	0
a ₂₁	a ₂₂	0	0
a ₃₁	a ₃₂	a ₃₃	0
a ₄₁	a ₄₂	a ₄₃	a ₄₄

we are at here (at the end)

we are at here (actually after a₄₂) [after subtracting]

$$\begin{aligned} &= 1000 + (10 - (2+1) + 1) \times 2 \\ &= (1000 + 8) \times 2 \\ &= 1016 \end{aligned}$$

$$\frac{4 \times 5}{2}$$

$$\begin{aligned} \text{Loc}(a[4][1]) &= 1000 + \left(\frac{4 \times 5}{2} - (4-1+1) \text{NNS} + (4-1) \right) \times 2 \\ &= 1000 + (10 - 10 + 3) \times 2 \\ &= 1000 + 6 = 1006 \end{aligned}$$

int * a = 2;
pf(+a);

$A[lb_1, ub_1, lb_2, ub_2]$

skip all
columns

go to
the starting
of the desired
column

$$LOC(A[i][j]) = B.A + ((ub_2 - lb_2) - (ub_2 - j + 1)NNS + (i - j)) \times C$$

go to
the desired
row

Q. $A[5, \dots, +5, -5, \dots, +5]$
 $BA = 1000$, $C = 10$
 Lower Triangular Matrix
 Column Major Order

$$\frac{66 - 21}{2} = 45$$

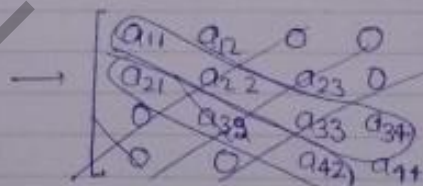
$$\begin{aligned} LOC(A[0][0]) &= 1000 + \left(\frac{11 \times 12^6}{2} - \frac{36 \times 7}{2} + (0 - 0) \right) \times 10 \\ &= 1000 + 45 \times 10 \\ &= 1450 \end{aligned}$$

$$\frac{7 \times 8 - 21}{2} = 24$$

$$\begin{aligned} LOC(A[2][1]) &= 1000 + (11NNS - (5 - (-1) + 1)NNS + (2 + 1)) \times 10 \\ &= 1000 + (66 - 28 + 3) \times 10 \\ &= 1000 + 41 \times 10 \\ &= 1410 \end{aligned}$$

Tri-diagonal Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$



$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

$N-1 + N + N-1 = 3N-2$ memory space.

★ In Tridiagonal Matrix, apart from 1st & last row (which has 2 elements), rest all other rows have 3 elements.

a. Using Row Major :-

a_{11}	a_{12}	a_{21}	a_{22}	a_{23}	a_{32}	a_{33}	a_{34}	a_{43}	a_{44}
1000	02	04	06	08	10	12	14	16	18

$$\text{Loc}(a[3][4]) = 1000 + (3 \times (2+3) + (4-2)) \times 2$$

$$= 1000 + (5+2) \times 2$$

$$= 1014$$

assuming each row contains 3 elements, skipping desired no. of rows

but 1st row contains only 2 elements, subtracting 1 element

$$\boxed{\text{Loc}(a[i][j]) = B.A + (3 \times (j - lb_1) - 1 + (i - j + 1)) \times C}$$

$$\text{Loc}(a[4][4]) = 1000 + (3 \times (4-1) - 1 + (4-4+1)) \times 2$$

$$= 1000 + 3 \times 2 = 1018$$

→ going to the desired column.

Using Column Major

$$\boxed{\text{Loc}(A[i][j]) = B.A + (3 \times (j - lb_2) - 1 + (i - j + 1)) \times C}$$

$$\text{Loc}(A[3][4]) = 1000 + (3 \times (4-1) - 1 + (3-4+1)) \times 2$$

$$= 1000 + (8) \times 2$$

$$= 1016$$

Using Diagonal by Diagonal :- (Principal diagonal, Lower, Upper)

				principal				Lower			Upper		
a_{11}	a_{12}	a_{13}	a_{14}	a_{11}	a_{22}	a_{33}	a_{44}	a_{21}	a_{32}	a_{43}	a_{12}	a_{23}	a_{34}
a_{21}	a_{22}	a_{23}	a_{24}										
a_{31}	a_{32}	a_{33}	a_{34}										
a_{41}	a_{42}	a_{43}	a_{44}										

$\text{Loc}(A[i][j]) \rightarrow$ if $(i=j)$ (Principal diagonal)

$$= B.A + (j - lb_1) \times C \text{ [if } i=j]$$

$$= B.A + ((ub_2 - lb_2 + 1) + j - lb_2) \times C \text{ [if } i > j]$$

$$N(ub_2 - lb_2 + 1 = N)$$

$$= BA + (N + (N-1) + (i - lb_1)) \times C \quad (\text{if } i < j)$$

Z-Matrix

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

Storing :-

- ① Row Major
- ② Column Major
- ③ Store 1st row, last row & remaining elements in diagonal.

Row-Maj

Using 3rd approach -

a_{11}	a_{12}	a_{13}	a_{14}	a_{41}	a_{42}	a_{43}	a_{44}	a_{23}	a_{32}
↓ N				↓ N				↓ N-2	

total space = $3N-2$

$$\text{Loc}(A[i][j]) = \begin{cases} BA + ((j - lb_2)) \times C & [\text{if } i = lb_1] \\ BA + (N + (j - lb_2)) \times C & [\text{if } i = ub_1] \\ BA + (2N + (i - lb_1 - 1)) \times C & [\text{if } j \neq lb_1 \text{ \& } j \neq ub_1 \text{ \& } (i+j) = N+1] \end{cases}$$

Tochitz Matrix

if $A[i,j] = A[i-1,j-1] \quad \forall i,j$ where $i > 1 \text{ \& } j > 1$

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

10	20	30	40
50	10	10	50
20	50	10	20
70	60	50	10

Store only $(2N-1)$ elements

Store only these

Storing:-

- ① Row-Order
- ② Column-Order
- ③ Store 1st row, remaining element in 1st column

1000	a_{11}	a_{12}	a_{13}	a_{14}	a_{21}	a_{22}	a_{23}
	10	20	30	40	50	60	70
	00	02	04	06	08	10	12

$$\text{Loc}(a[i][j]) = \begin{cases} B.A + (j - lb_2) \times C & \text{(if } i \leq j) \\ B.A + (N + (i - lb_1 - 1)) \times C & \text{(if } i > j) \end{cases}$$

[Elements belong to upper part]
[Elements belong to lower part]

Three-Dimensional Matrix

int a[2][3][4] → 2 arrays of size 3x4

BA = 1000

Row Major Order

$$\begin{aligned} \text{LOC}(A[2][3][4]) &= 1000 + ((2-1) \times 12 + (3-1) \times 4 + (4-1)) \times 2 \\ &= 1000 + (12 + 8 + 3) \times 2 \\ &= 1000 + 46 = 1046 \end{aligned}$$

Loc of A

int a[-5...+5][-10...+50][-25...+25]

• BA = 0

• Column Major Order

• C = 5

$$\begin{aligned} \text{Loc}(A[0][0][0]) &= 0 + (5 \times 61 \times 51 + (0+25) \times (61) + (0+10)) \times 5 \\ &= (3111 + 1525 + 10) \times 5 \\ &= 85450 \end{aligned}$$

$$\begin{array}{r} 3111 \\ \times 5 \\ \hline 15555 \\ 15555 \\ \hline 17090 \end{array} \quad \begin{array}{r} 3 \\ \times 61 \\ \hline 183 \\ 1830 \\ \hline 17090 \end{array}$$

$$A[lb_1 \dots ub_1][lb_2 \dots ub_2][lb_3 \dots ub_3]$$

Row Major Order

$$LOC(a[i][j][k]) = B.A + ((i-lb_1) \times (ub_2-lb_2+1)(ub_3-lb_3+1) + (j-lb_2) \times (ub_3-lb_3+1) + (k-lb_3)) \times C$$

Column Major Order

$$LOC(a[i][j][k]) = B.A + ((i-lb_1) \times (ub_2-lb_2+1)(ub_3-lb_3+1) + (k-lb_3) \times (ub_2-lb_2+1) + (j-lb_2)) \times C$$

specifying this is optional

Q. $\text{int } a[5] = \{10, 20, 30, 40, 50\};$

$\text{int } a[5]; \rightarrow$ garbage values

$\text{int } a[5] = \{10, 20, 30\};$

4th & 5th elements are 0.

$\text{int } a[][3] = \{10, 20, 30, 40, 50, 60\};$

optional

$\text{int } a[][2][3]$

optional

mandatory

$\text{int } a[5] = \{ \}; \rightarrow$ all are 0.

$\text{int } a[5] = \{10, 20, 30, 40, 50, 60\};$

no compilation error, but $a[5]$ is not accessible

Note:-

In multidimensional Array, M-1 indexes are mandatory.

Q. $\text{main}()$

$\{ \text{int } a[3][3];$

$\text{printf}("%d", ((a=*a)++ (*a = a[0])));$

$a = 1000$

10	20	30
40	50	60
70	80	90

$a = 1000$

$a+0 = 1000 \Rightarrow *(a+0) = *1000 = 1000$

$a+1 = 1006$

$a+2 = 1012$

$*(a+1) = *1006 = 1006$

$*(a+2) = *1012 = 1012$

row is not selected

row is $*a+1 = a[1]$ selected.

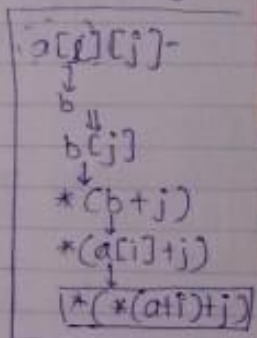
row is not selected & hence we can change the row.

$(a+1) = 1006$

$(a+1)+1 = 1012$

$a = 1000$
 $a+1 = 1006$

$*(a+1) = 1006 \rightarrow$ row is selected (we can't change the row now.)



- $a = 1000$
- $a+1 = 1006 \rightarrow$ row is not selected
- $*(a+1) = 1006 \rightarrow$ row is selected
- $*(a+1)+1 = 1008$
- $*(*(a+1)+1) = 50$
- $*(*(a+1)+1)+1 = 51$

Q. main()

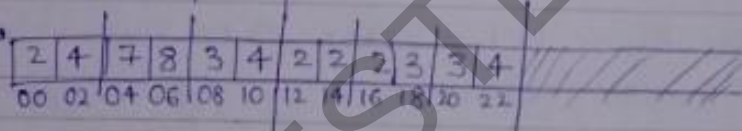
```

{ int a[2][3][2] = {2,4,7,8,3,4,2,2,2,3,3,4};
  printf("%u %u %u %d", a, *a, **a, ***a);
  printf("%u %u %u %d", a+1, *a+1, **a+1, ***a+1);
  printf("%u %u %u %d", *(*(a+2)+3), *(*(a+1)+2), *(a+2), *(*(a+1)+1));
  printf("%u %u %u %d", a+1, a[1][1], a[1][2]+1, a[1][2][3]+1);
}

```

$a[2][3][2]$ -

$a = 1000$



$*a \rightarrow$ array selected
 $**a \rightarrow$ array & row selected
 $***a \rightarrow$ array, row & column selected

O/P:-

1000	1000	1000	2
1012	1004	1002	3
1036	2	1024	2
1012	1016	1022	garbage

3rd printf:-

$*(*(a+1)+2)$
 $\downarrow$
 $*(**a+2)$

$*a \rightarrow$ 0th array base address
 $**a \rightarrow$ 0th array 0th row base address

$*(a+2)$

$*a = 1000$
 $\downarrow$
 array selected

$*(*(a+1)+1)$

$*(**a+1)$
 $\downarrow$
 array selected

array selected

skip 1 row = $1012 + 4 = 1016$

row selected, column selected

$*(a+1)+2$

$1012 + 2 \times 2$

$= 1016$

$*(1016)$

column selected

$*(*(a+2)+3)$

$1000 + 3 \times 2$

$= 1006$

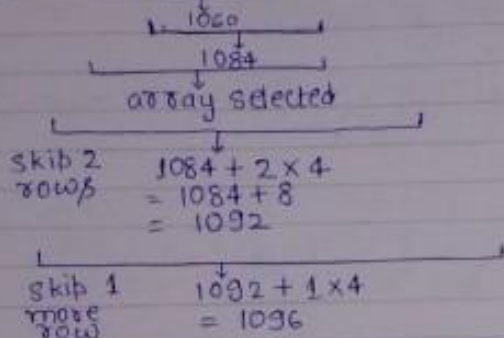
array selected

row $\rightarrow 1024 + 3 \times 2 = 1032$

row selected

$$12 \times 7 = 84$$

$$Q \rightarrow ((*(a+5)+2)+2)+1$$



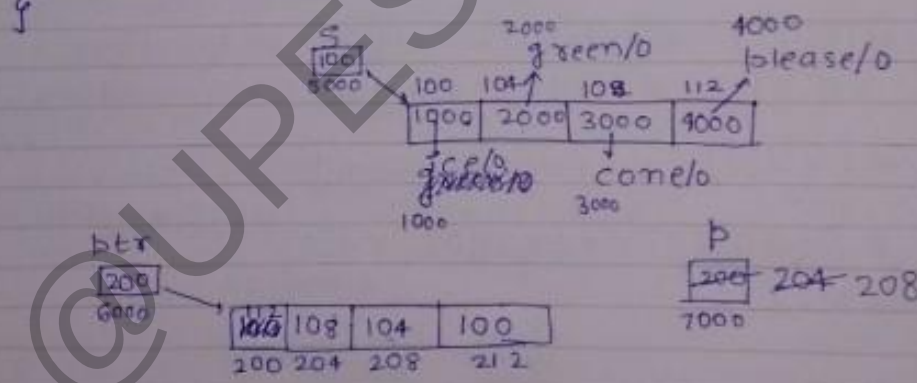
#include <stdio.h>

Q main()

```

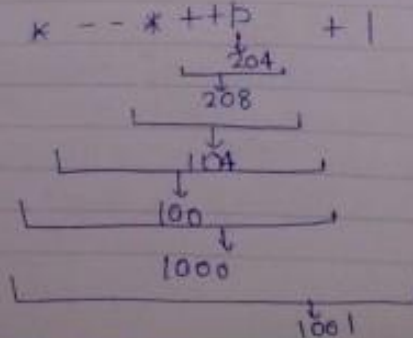
{ char *s[] = {"ice", "green", "cone", "please"};
  char **ptr[] = {s+3, s+2, s+1, s};
  char ***p = ptr;
  printf("%s", **++p);
  printf("%s", *--*++p+1);
}

```



O/p:-

$b = 200$
 $++p = 204$
 $*++p = 3000$
 $**++p = \text{cone}$



$\therefore$ O/p:-
 cone
 ce

Q #include <stdio.h>

int a1[] = {6, 7, 8, 34, 67};

int a2[] = {25, 56, 28, 29};

int a3[] = {-12, 27, -31};

int *x[] = {a1, a2, a3};

main()

{ B(x);

}

B(int **a)

{ printf("%d", a[0][2]);

printf("%d", *a[2]);

printf("%d", **a++ + a[0]);

printf("%d", *(++a)[0]);

}

O/p :-

1st printf :- a[0][2] = ~~*(a+0)+2~~ = 1002

1st printf :- a[0][2] = ~~*(a+0)+2~~ = ~~*(1000+2*2)~~ = ~~*(1004)~~

2nd printf :- *a[2] = ~~*(a+2)~~ = ~~*(4000+2*2)~~ = ~~*(4004)~~ = ~~*(3000)~~

3rd printf :- *++a[0] = ~~*(a+0)~~ = ~~*(4000)~~ = ~~*(1000)~~ = ~~*(1002)~~

4th printf :- ~~*(++a)[0]~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

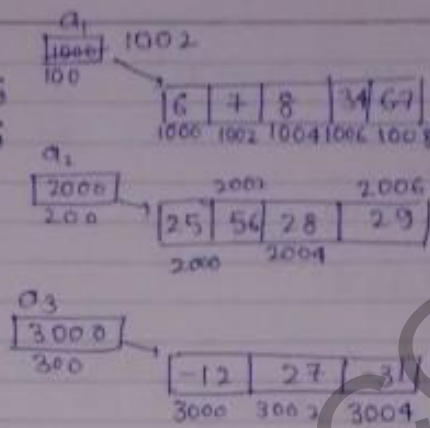
~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~

~~*(++a)~~ = ~~*(a+0)~~ = ~~*(a[0])~~ = ~~*(1002)~~



O/p :-
1002
25
7
25

even though

1-D pointer array.

★ using one dimensional array of pointer, we can get two-dimensional array.

int a, *b

★ int a[5] → 2-D array, where every row contain uniform no. of columns.

★ int *x[] → 2-D array, but it will create variable columns in every row.

★ int a₁[] = {6, 7, 8, 34, 67};
int a₂[] = {25, 56, 28, 29};
int a₃[] = {-12, 27, -31};
int a₄[] = {10, 20};
int a₅[] = {20};

int *x[] = {a₁, a₂, a₃, a₄, a₅};



Q. consider following C-declarations:-

(1) int A[10][10];

(2) int *B[10];

Which is true?

(a) A[5][6] = 10; T

(b) A[5] = 1000 F (we can't change the base address of 5th row of an array.)

(c) B[5][6] = 10; T

(d) B[5] = 1000 T

Q. main()

{ char string[] = "Hello World";

display(string);

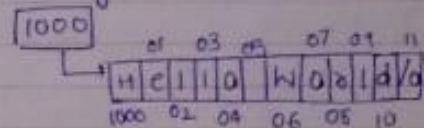
}

void display(char *string)

{ printf("%s", string);

}

String



★ In the above program, we are using display fn. w/o declaration so compiler assumes display fn by default integer value, but above program is returning void, so above program will give error of type mismatch.

Q. double foo(double); /* line 1 */
int main()

```
{ double da, db;
  db = foo(da);
  return a;
double foo(double a)
{ return a;
}
```

* If we delete line 1 then
conflicting datatype for fn. foo.
type mismatch

* In switch statement, we
can write default wherever
we want, but it will be
executed whenever there is
no match.

Q. void main()
{ int i=0;
for(int i=0; i<20; i++)

{ switch(i)

```
{ case 0: i+=5; // we can
  case 1: i+=2; put default
  case 5: i+=5; anywhere,
  default: i+=4; he will have
    break; some result.
```

{ printf("%d", i); if default is
at *

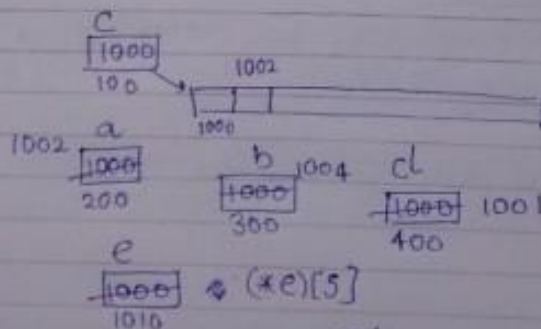
}

O/p:-

5
16
27
32
37
42
47
52

i=i+5
5

Q. main()
{ int c[100];
int *a = (int *)c;
float *b = (float *)c;
char *d = (char *)c;
int (*e)[5] = (int *)c;
for(i=0 to 5)
{ printf("%d", a[i], b[i], d[i], e[i]);
a++;
b++;
d++;
e++;
}



O/p:-

i	a[i]	b[i]	d[i]	e[i]
0	1000	1000	1000	1000
1	1002	1002	1004	1001
2	1004	1004	1008	1002
3	1006	1006	1012	1003
4	1008	1008	1016	1004
5	1010	1010	1020	1005

★ $\text{int} (*a)[5] \rightarrow$ a is a pointer to array which contains 5 elements,
 when we do $++$, then a is incremented to 10B.

Function returning pointer :-

```
int fun();
main()
{
    int x=10;
    int *b;
    b = fun();
    printf("%d", x+(*b));
}
```

x
 1000
 10
 b
 3000
 2000

```
int *fun()
{
    int y=20;
    return (&y);
}
```

y
 20
 3000

★ function will get the memory when the fn. is called, & will be deallocated when the fn. returns.

O/p:-
 10+20=30

★ fun is a function which returns pointer.

return int a;
 static
 int x=20;

Functional Pointer

```
main()
{
    int display(); //creating memory for display
    int (*funptr)(); //fn (base address is allocated)
    funptr = display;
    (*funptr)();
}
```

```
int display()
{
    printf("Hi");
    return (0);
}
```

★ function name indicates base address.
 display indicates its base address.

value at 1000 (at 1000, it is the fn. display)
 funptr is a pointer, which internally contain function's address.

$funptr$
 1000
 2000
 $display$
 1000
 3000

O/p:-
 Hi

★ `int (*p)()` → `p` is pointer pointing to a function.

`int *p[5]` → `p` is an array of pointers pointing to

`int *p[5]()` → `p` is an array of pointers pointing to 5 fn. which are returning integer

Q. `main()`

```
{ int (*ptr[3])();
```

```
  ptr[0] = aaa;
```

```
  ptr[1] = bbb;
```

```
  ptr[2] = ccc;
```

```
  ptr[2](); or (*ptr[2])();
```

```
int aaa() { pf("aaa");
```

```
  return 0;
```

```
}
```

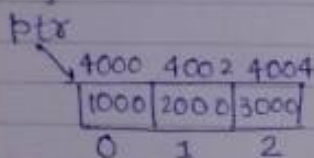
```
int bbb() { pf("bbb");
```

```
}
```

```
int ccc() { pf("ccc");
```

```
}
```

ptr



`*ptr[0]` → `** (ptr + 0)` `ptr[0] = *(ptr + 0)`
 = `** (1000)` = `*(4000)`

• value at address 4000

`(*ptr[2])()`

↓
`(** (ptr + 2))()`

↓
`(** (4004))()`

↓
`(*(3000))()`

at ccc fn.

Q. What does the following C-statement indicate

```
int (*f)(int, int);
```

★ `f` is a pointer containing address of a function which is taking two int variables as argument & returning an integer

• `int * (*f)(int *, int *);`

`f` is a pointer containing address of a function which takes 2 integer pointers as arguments & returns integer pointer.

② `void (*abc)(int, void (*def)());`
 abc is a pointer containing address of a function which takes 2 arguments as i/p, out of which one is integer & the other is ② def is a pointer pointing to a fn which returns void & returns void.

③ `char * (*(*a[]))();`
 Array of N pointers pointing to function(s) [doesn't take any input]
 these function(s) are returning pointer(s) (pointer type is not specified)
 these returned pointers are pointing to a function
 this function is returning character pointer.

* Array of N pointers pointing to functions which will return a pointer which is pointing to a function which is returning character pointer.

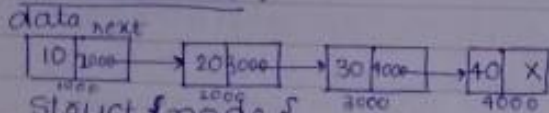
④ `(char * (*)) (*ptr[N])();`
 (i) a pointer is pointing to a function
 (ii) the function returns a character pointer.
 (iii) character pointer is pointing to an array of pointers which are pointing to functions which are returning a pointer.

brackets are having associativity from left to right

Date

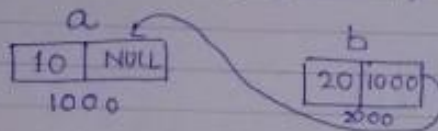
12.08.12

Linked List :-



```
struct node {
    int data;
    struct node *next;
};
```

```
struct node a = {10, NULL};
```



```
struct node b = {20, &a};
```

```
struct node *ptr = &b;
```

Self referential data structure

- float type pointer is pointing to float.
- int type pointer is pointing to int.

ptr = 2000

*ptr → inside 2000 (i.e. Selected the address 2000).

(*ptr).next = 1000

(*ptr).next :- Selected the address 1000
(*(ptr->next)).data = 10

↓
(*(ptr->next)).data

(ptr->next) → data = 10

Q Write a C program to print all data fields present in every node of the linked list.

Ans.

```
#include <stdio.h>
```

```
struct
```

```
main()
```

```
{ struct node *start = //starting address of linked list
```

```
while
```

```
{ ptr = start;
```

```
while (ptr != NULL)
```

```
{ printf("%d", ptr->data);
```

```
ptr = (*ptr).next;
```

```
}
```

```
}
```

```
int a;
int *ptr = &a;
ptr = (int*) 0;
```

*if s is NULL, then $*s$ is not allowed (it gives segmentation error)

Q. Write a C program to return data of the last node of linked list

Ans. `ptr = start;`
`while (ptr != NULL)` // Linked list contain atleast 1 node.
`{ if ((ptr->next) == NULL)`
`{ printf("%d", (ptr->data));`
`}`
`ptr = (ptr->next);`
`}`

* struct node {

int data;
 struct node *next;

};

int Display(struct node *s)

{ struct node *p = s;

if (p == NULL)

{ printf("Linked list is empty");
 exit(0);

}

while (p->next != NULL) // Linked list contain atleast 2 elements

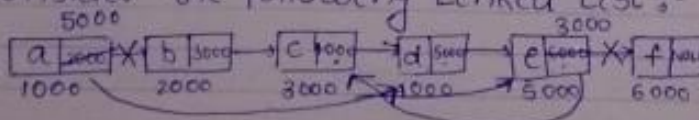
{ p = p->next;

}

printf("%d", p->data);

}

Q. Consider the following Linked List :-



What would be the O/P after executing following sequence of statements :-


```

int a = 10;
int b = 20;
printf("a * b");

```

p
3000

first
1000

```

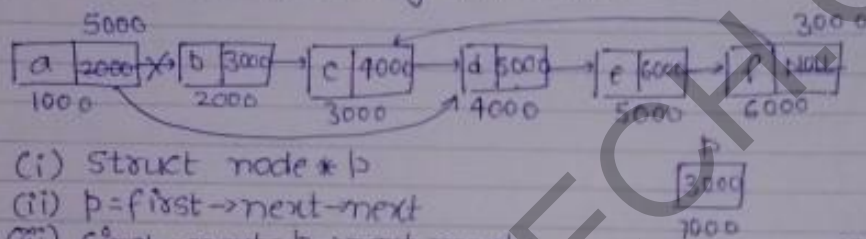
struct node *p;
p = first->next->next;
first->next = p->next->next;
p->next->next->next = p;
printf("first->next->next->next->data");

```

O/P:-

a

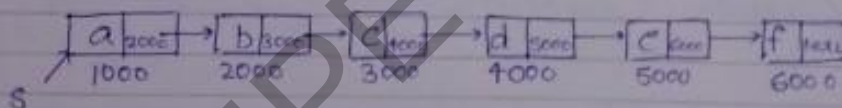
Q. Consider the following linked list -



- struct node *p
- p = first->next->next
- first->next = p->next->next
- p->next->next->next->next = p
- printf("first->next->next->next->data");

O/P:-
c

Adding Node in Linked List



(i) Starting of the linked list -

```

Add at Start (struct node *s, int x)
{
    struct node *ptr = (node *) malloc (sizeof (node));
    (*ptr).next = s;
    (*ptr).data = 'g';
    s = ptr;
}

```

(ii) Write a C program to add a node with data x at the end of the linked list.

```

Addatend (node * start)
{
    node * ptr = (node *) malloc (sizeof (node));
    (*ptr).data = x;
    node * temp = start;
    while (ptr->next != NULL)
    {
        temp = temp->next;
    }
    temp->next = ptr;
    ptr->next = NULL;
}

```

(iii) Write a C program to add a node with data x, after a node with data y.

```

→ Addinbtw (node * start)
{
    node * ptr = (node *) malloc (sizeof (node));
    (*ptr).data = x;
    node * temp = start;
    while (temp->data != y)
    {
        temp = temp->next;
    }
    if (temp == NULL)
    {
        exit(0);
    }
    ptr->next = temp->next;
    temp->next = ptr;
}

```

(iv) Write a C program to add a node with data x before a node with data y.

```

Addbefore (node * start)
{
    node * ptr = (node *) malloc (size of (node));
    (*ptr).data = x;
    node * temp = start;
    node * tempbef = start;
    while (temp->data != y)
    {
        tempbef = temp;
        temp = temp->next;
    }
    if (temp == NULL) exit(0);
}

```



```

if (tempbef == start)
{
ptr ptr->next = start;
start = ptr; exit(0);
}
temp->next = ke
ptr->next = tempbef->next;
tempbef->next = ptr;
}
Or
Addbef (struct node *start)
{
node *p = (node *) malloc (sizeof (node));
p->data = x;
node *q = start;
if (q == NULL)
return(1);
if (q->data == 'y')
{
p->next = q;
start = p;
}
while (q->next != NULL && (q->next)->data != 'y')
{
q = q->next;
}
if (q->next == NULL)
exit(1);
q->next p->next = q->next;
q->next = p;
}

```

Q. Which one of the following statement, adding an element x after current position

- current = Newnode(x, current)
- current = Newnode(x, current->next);
- current->next = Newnode(x, current);
- ☒ current->next = Newnode(x, current->next);

Q Consider the fn. f defined below:-

```
struct item
{
    int data;
    struct item *next;
};
int f(struct item *p)
{
    return((p==NULL) || (p->next==NULL) || ((p->data <= p->next->data)
        && f(p->next)));
}
```

for a given LL, the fn. f returns 1 iff

(a) if LL is empty or exactly one element

(b) elements in the LL are sorted in non-decreasing order.

(c) " " " " " " " " -increasing "

(d) none of the above.

indirectly means
if LL has 0 or 1 element
then it is also in non-
increasing order.

Q Write a C program to delete a node at the beginning of the linked list.

```
* del(node *start)
{
    node * ptr = start;
    start = (*start).next;
    free(ptr); // the memory is deallocated pointed by
               // ptr, now *ptr is dangling pointer, so to avoid
    ptr = NULL; // any segmentation errors, we perform ptr = NULL.
}
```

* if we write int a, then we can't deallocate the memory using free().

Q Write a C program to delete a node at the end of LL.

```
delatEnd(node *start)
{
    node * ptr = start; prev = start;
    if (ptr == NULL)
        exit(1);
}
```


at=a+b;

```
while (ptr->next != NULL)
{
    prev = ptr;
    ptr = ptr->next;
}
if (prev == start)
{
    free(prev);
    start = prev = ptr = NULL;
}
else
{
    prev->next = NULL;
    free(ptr);
    ptr = NULL;
}
}
```

2. Write a C program to delete a node before which contains data x.

```
del (struct node * start)
{
    node * ptr = start, * prev = start;
    if (ptr == NULL)
    {
        exit(1);
    }
    while (ptr->data != 'x' && ptr->next != NULL)
    {
        prev = ptr;
        ptr = ptr->next;
    }
    if (ptr == NULL) { exit(1); }
    if (prev == ptr == start)
    {
        start = start->next;
        free(ptr);
        ptr = NULL;
    }
    else
    {
        prev->next = ptr->next;
        free(ptr);
        ptr = NULL;
    }
}
```

Q Write a C program to delete a node before x.
Using 3 pointers

```
del (node * start)
{
    node * ptr = start, * prev = start, * prevtwobef = start;
    if (ptr == NULL)
        exit(1);
    while (
        if (start->data == 'x')
            exit(0);
        if (start->next->data == 'x')
        {
            start = start->next;
            free(ptr);
            ptr = prev = prevtwobef = NULL;
        }
        while (
            ptr = (start->next)->next;
            prev = start->next;
            prevtwobef = start;
            while (ptr->data != 'x' && ptr->next != NULL)
            {
                prevtwobef = prev;
                prev = ptr;
                ptr = ptr->next;
            }
            if (ptr->next == NULL && ptr->data != 'x')
                exit(1);
            prevtwobef->next = prev->next;
            free(prev);
            prevtwobef = prev = ptr = NULL;
        }
    }
```

Or

Or

Using 2 pointers

```
del (node * start)
{
    node * ptr = start, * prev = start;
    if (start == NULL)
        {exit(1);}
    if (start->data == 'x')
        {exit(1);}
    if (start->next->data == 'x')
        {
            start = start->next;
            prev = ptr = NULL;
            {exit(0);}
        }
    while (ptr->next)
        ptr = start->next->next;
    prev = start->next;
    while ((ptr->next->data != 'x' && (ptr->next) != NULL))
        {
            prev = ptr;
            ptr = ptr->next;
        }
    if ((ptr->next) == NULL && ptr->data != 'x')
        {
            {exit(1);}
        }
    if (& prev->next = ptr->next;
        free(ptr);
        ptr = prev = NULL;
    }
}
```

Using 1 pointer

```
del (node * start)
{
    node * ptr = start;
    if (start == NULL)
        {exit(1);}
    if (start->data == 'x')
        {exit(1);}
}
```

Note :- In order to delete any node in the given linked list apart from last node two pointers (minimum 2 pointers) are required.

Q. Write a program to find middle of the linked list

```
find(node *start)
{
    node * ptr = start;
    int i = 0;
    while (ptr->next != NULL)
    {
        i++;
        ptr = ptr->next;
    }
    if (ptr == NULL)
    {
        exit(1);
    }
    if (ptr == start)
    {
        return (ptr);
    }
    if (i % 2 == 0)
    {
        i = i / 2;
    }
    else
    {
        i = i / 2 + 1;
    }
    ptr = start;
    for (int j = 1; j <= i; j++)
    {
        ptr = ptr->next;
    }
}
```

Using 2 loops

$O(n)$

Using 1 loop

struct node

```
find(node *start)
```

```
{
    node * ptr = start, * ptrmid = start;
```

```
    while (ptr->next != NULL && (ptr->next->next != NULL))
```

```
    {
        ptrmid = ptrmid->next;
```

```
        ptr = (ptr->next->next);
```

```
    }
    return (ptrmid);
```

```
}
```

$O(n)$

$\frac{n}{2}$ times

* Binary Search can be applied on Linked List but it is not efficient because it will take $O(n)$ times + $\log_2 n$ extra space.

Q Write a C program to perform Binary Search on LL.

```
node * BS(node * start, int x)
{
    node * ptr = start;
    node * ptomid = start;
    node * m;
    while (ptomid->data)
    {
        if (start->next == NULL)
        {
            if (start->data == x)
                return (start);
            else
                return (NULL);
        }
        else
        {
            m = findmid(start); //  $O(n)$ 
            if (m->data == x) //  $O(1)$ 
            {
                return (m);
            }
            else if (m->data < x) //  $O(1)$ 
            {
                m->next = NULL;
                BS(start);
            }
            else
            {
                start = m->next;
                BS(start);
            }
        }
    }
}
```

Let $T(n)$ be the time complexity of Binary Search algo on a linked list which contains n elements, then

$T(n) = O(1)$, if $n=1$

$n + T(\frac{n}{2})$, if $n > 1$

$\downarrow$
 $O(n)$

$\left[\begin{array}{l} \log_2 n \\ \text{stack space} \\ \text{is req} \end{array} \right]$

$$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$$

Q Write a C program to sort the given linked list.

BubbleSort (node *start)

```
{ node *ptr = start;  
  node *prev = start;
```

```
  for (int i
```

```
    int i = 1;
```

```
    while (ptr->next != NULL)
```

```
      { ptr = ptr->next;
```

```
        i++;
```

// length of linked list

```
      { ptr = start->next;
```

```
        for (int j = 0; j < i; j++)
```

```
          { while (ptr->next != NULL)
```

```
            { if (prev->data > ptr->data)
```

```
              { prev->data = ptr->data;
```

```
                prev->data = ptr->data;
```

```
                ptr->data = temp;
```

```
            }
```

```
            prev = ptr;
```

```
            ptr = ptr->next;
```

```
          } ptr = start;
```

```
        }
```

```
      }
```

```
    }
```

08

Sort (node *start)

```
{ node *ptr = start;
```

```
  int i = 1;
```

```
  while (ptr->next != NULL)
```

```
    { ptr = ptr->next;
```

```
      i++;
```

```
    }
```

```
    ptr = start;
```

```
    for (int j = 0; j < i; j++)
```

```
      {
```



```
while (ptr->next != NULL)
```

Or

```
Sort (struct node * s)
{ struct node * p, * q;
  for (p = s; p != NULL; p = p->next)
  { if (p->data
    for (q = p->next; q != NULL; q = q->next)
    { if (p->data > q->data)
      { swap(p->data, q->data);
      }
    }
  }
}
```

Q. Write a C-program to reverse the given LL.

```
Reverse (node * start)
{ node * ptr = start; node * temp1, node * temp2;
  int i = 1; int temp1 = start;
  while (ptr != NULL) temp2 = start->next;
  while (ptr != NULL)
  { ptr = ptr->next;
    temp2 = (ptr->next)->next;
    ptr->next = temp2;
    (ptr->next)->next = temp1;
    temp1 = ptr->next;
    temp2 = temp2->next;
  }
}
```

~~node~~

```
struct node1  
{ struct node * addr; struct node * next;  
};
```

Reverse (struct node * start)

```
{ node * ptr = start; node1 * ptr1;  
while (ptr != NULL)  
{ ptr1 = (node1 *) malloc (sizeof (node1));  
ptr1->addr = ptr;  
ptr = ptr->next;  
}  
ptr1->ptr = start;
```

Or

Reverse (struct node * s)

```
{ struct node * p, * q, * r;  
r = s; q = p = NULL;  
while (r != NULL)  
{ p = q;  
q = r;  
r = r->next;  
q->next = p;  
}  
S = q;
```

→ 3 pointers compulsory.

$O(n)$

- Q The following C func. takes a single LL as i/p argument. It modified the list by moving the last node to the front of the list & returns the list. Some part of the code is left blank.


```

typedef struct node {
    int value;
    struct node *next;
} node,

```

```

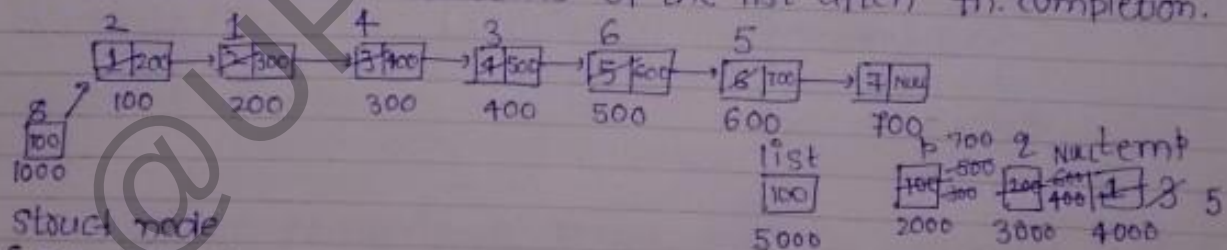
node * modifiedlist (node * head)
{
    node * p, * q;
    if ((head == NULL) || (head->next == NULL))
        return (head);
    q = NULL;
    p = head;
    while (p->next != NULL)
    {
        q = p;
        p = p->next;
    }
    p->next = head;
    q->next = NULL;
    head = p;
    return (head);
}

```

Q. The following C fn. takes a singly linked list of integers as parameter & rearranges the elements of list. The fn. is called with the list containing the integers -

1, 2, 3, 4, 5, 6, 7

What will be the contents of the list after fn. completion.



```

struct node
{
    int value;
    struct node *next;
};

```

```

void arrange (struct node *list)
{
    node *p, *q;
    int temp;
    if (!list || !list->next)
        return;
    p = list;
    q = list->next;
    while (q)
    {
        temp = p->value; p->value = q->value; q->value = temp;
        p = q->next;
        q = p->next;
    }
}

```

list now :- 2, 1, 4, 3, 6, 5, 7

Q. Write a C program to find a cycle (loop) in the given linked list.

```

struct node
{
    node * addr;
    node * next;
};

```

```

findcycle (node * start)
{
    node * start1 = (node *) malloc (sizeof (node));
    start1->addr = start;
    start1->next = NULL;
    node * prev = start1;
    node * ptr = start;
    while (ptr != NULL)
    {
        node * ptr1 = (node *) malloc (sizeof (node));
        prev->next = ptr1;
        prev = ptr1;
        ptr1->addr = ptr; ptr = ptr->next;
        ptr1->next = ptr;
    }
}

```


Or

Algorithm

- ① Take two pointers p_1 & p_2 .
- ② increment p_1 by 1 & p_2 by 2.
- ③ Max. n loops p_1 & p_2 are same comes out to be same. When p_1 & p_2 comes out to be equal, then there's a cycle.

Algorithm:-

Store every address in Array if that is not found in array continue until match is found.

* If we allocate memory using Malloc, then elements inside struct, values have garbage value.

* If we allocate memory using calloc, then elements inside struct will be initialized to 0.

Q Write a C program to implement Stack using LL.

Q Write a C " " " " Queue " "

Q " " " " " " LL " arrays

Q " " " " " perform following ops on LL:-

(i) Union of 2 LL.

(ii) Intersection of 2 LL.

(iii) Polynomial addition of 2 LL.

(iv) " multiplication of 2 LL.

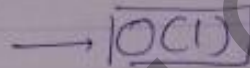
Date

18.08.12

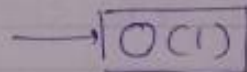
Stacks

```
struct stack { int data;  
                struct stack *next;  
}; struct stack *top = NULL;
```

```
Push(struct stack *top, int x)  
{  
    struct stack *p = (struct stack *) malloc(sizeof(struct stack));  
    if (p == NULL)  
    { printf("Memory is full");  
      exit(1);  
    }  
    else  
    { p->data = x;  
      p->next = top;  
      top = p;  
    }  
}
```



```
Pop(int x, struct stack *top)  
{ struct stack *temp = top;  
  if (top == NULL)  
  { printf("Stack is empty");  
    exit(1);  
  }  
  top = temp->next;  
  x = temp->data;  
  free(temp);  
  temp = NULL;  
}
```



Queue

★ If To implement Queue properly, we need atleast 2 pointers (Front & Rear)

Add

```
struct queue {  
    int data;  
    queue * next;  
};
```



```
struct queue *front = NULL, *rear = NULL;
```

```
Add(int x)
```

```
{ queue *temp = (queue *) malloc (sizeof (queue));
```

```
temp->data = x;
```

```
if (front == NULL && rear == NULL)
```

```
{ front = temp;
```

```
rear = front;
```

```
front->next = NULL;
```

→ [OCI]

```
{  
    {  
        elseif (front == rear)
```

```
{ if temp->next = front; rear->next = temp;  
rear = front = temp;
```

```
}
```

```
Del(int x)
```

```
{ if queue *temp = front;
```

```
if (front == rear == NULL)
```

```
{ printf("Queue is empty");
```

```
exit(1); // no element
```

```
}
```

```
if (front == rear)
```

→ [OCI]

```
{ front = front->next;
```

```
rear = NULL;
```

```
free(temp);
```

// only one-element

```
temp = NULL;
```

```
}
```

```
else
```

```
{ front = front->next;
```

```
free(temp);
```

```
temp = NULL;
```

```
}
```

```
}
```

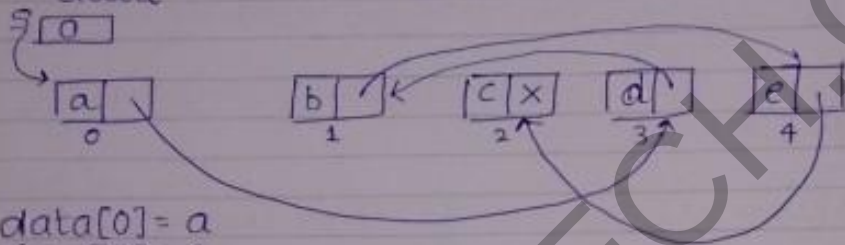
Implementing Linked List Using Arrays

0	a
1	b
2	c
3	d
4	e

data

0	3
1	4
2	-1
3	1
4	2

link



data[0] = a
link[0] = 3

Q. Write a C program to display data part of the linked list using array representation.

```
display(int *d, int *l, int *start)
{
    int *temp = start;
    while (*temp != -1)
    {
        printf("%d", data[temp]);
    }
}
```

```
display(data, link, s)
{
    int temp = s;
    while (temp != -1)
    {
        printf("%d", data[temp]);
        temp = link[temp];
    }
}
```

Q. Write a C program to delete a ^{last} node from linked list

del(data, link, s)

{ temp = s;

prev = s;

while (temp != -1) link[temp] != -1)

{ prev = temp;

temp = link[temp];

}

link[prev] = -1;

}

or

deletelast(data, link, s)

{ temp = prev = s;

if (link[s] == -1)

{ s = -1;

exit(1);

}

else

{ while (link[temp] != -1)

{ prev = temp;

temp = link[temp];

}

link[prev] = -1;

}

}

Union Of 2 Linked List :-

P → 10 → 20 → 30 → 40 NULL

$O(n^2)$

Q → 15 → 20 → 25 → 30 NULL

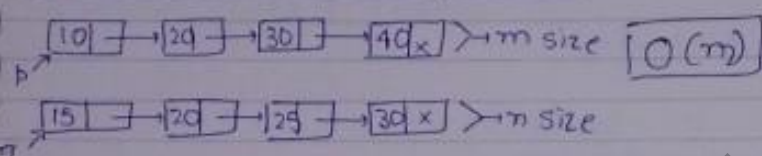
→ 10 → 20 → 30 → 40 → 15 → 25 NULL

Q/S

Algo:-

1. Add first linked list to the result. $\rightarrow O(n)$
2. Take every element of 2nd LL & check if it is available in first linked list or not using Linear Search.
 - if it is available, don't add it in LL. $\rightarrow O(n^2)$
 - otherwise add it to LL.

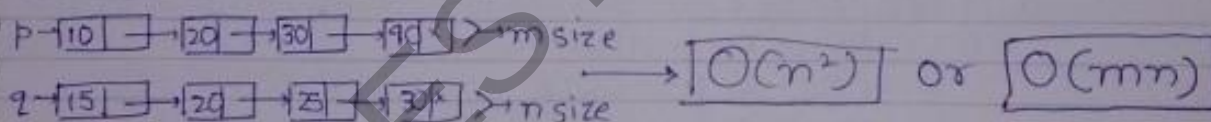
Concatenation of 2 LL



Algo:-

1. Go to the last node of the 1st LL. $\rightarrow O(n)$
2. & link that node with the starting node of the 2nd LL. $(O(1))$.

Intersection of 2 LL:-



1. Take the 1st node of 1st Linked List & check whether the node's data (of 1st LL) matches with any node's data of 2nd Linked List.
 - if it matches with ~~a~~ then add it to result LL.
 - otherwise don't add.
2. Repeat Step 1 with all the other nodes of 1st LL.

Polynomial Addition

$P: 10x^3 + 5x^2 + 2x + 5$

$Q: 20x^3 + 15x^2 + 5x + 20$

$30x^3 + 20x^2 + 7x + 25$

$P: 10x^{15} + 20x^5 + 3x^2 + 16$

$Q: 5x^5 + 77x + 10$

$10x^{15} + 20x^5 + 25x^5 + 3x^2 + 77x + 26$

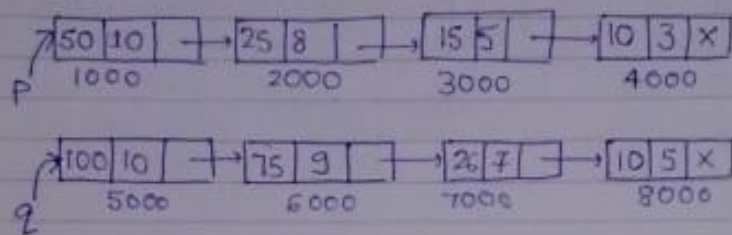

```

struct poly{
    int power;
    int coe;
    poly * next;
};

```

Polynomial addition (

Time complexity:-



$O(m+n)$

```

while(p != NULL && q != NULL)
{

```

```

    if (p->power == q->power)
    {
        r->power = q->power;
        r->coe = p->coe + q->coe;
        p = p->next;
        q = q->next;
        r = r->next;
    }

```

```

    else
    {

```

```

        if (p->power > q->power)
        {
            r->power = p->power;
            r->coe = p->coe;
            r = r->next;
            p = p->next;
        }

```

```

        else
        {

```

```

            r->power = q->power;
            r->coe = q->coe;
            r = r->next;
            q = q->next;
        }
    }

```

```

    if (p == NULL && q == NULL)
        return (r);

```

```

    else
    {

```

```

        if (p == NULL)
            add q to r;
        else
            add p to r;
    }

```

```

}

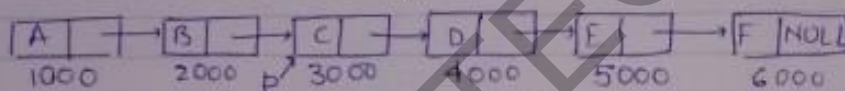
```

Polynomial Multiplication :-

```
while (p != NULL)
{
    while (q != NULL)
    {
        r->power = p->power + q->power;
        r->coe = p->coe * q->coe;
        r = r->next;
        q = q->next;
    }
    p = p->next;
    q = s; // starting of q
}
```

→ $O(n^2)$

Q. Consider the following LL:-



from the above linked list delete a node pointed by p if only p is given. (starting is not given.)

Ans.

```
del(Node *p)
{
    Node *temp = start;
    Node *prev = start;
    while (temp != p)
    {
        prev = temp;
        temp = temp->next;
    }
```

copy the content

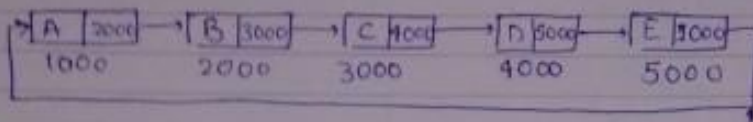
```
p->data = (p->next)->data; // but if p is last node,
p->next = (p->next)->next; // then this algo fails
free(p->next);
```

p

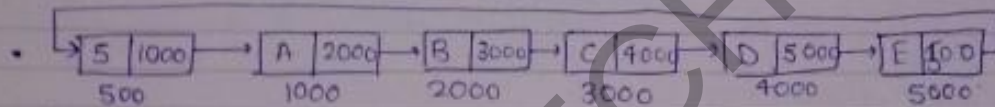
Drawbacks of Singly LL :-

- ① We can move only in one direction. (we can't go back)
- ② In singly LL last node's next part is always NULL w/o any use (we can keep starting node address, so that we can go back.)

Circular SLL :-



1. We can go back.
2. The drawback with circular SLL it may enter into the infinite loop.
3. In order to eliminate the infinite loop in circular SLL, we are going to head node.
4. Head node data part will contain valuable info. like total no. of nodes in circular SLL.

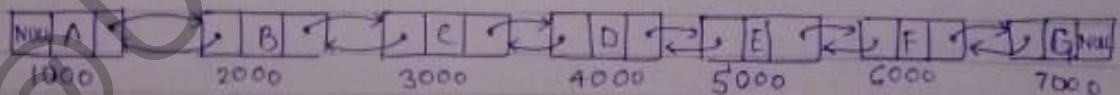


if

Doubly Linked List

struct DLL

```
{ struct DLL *left;  
  int data;  
  struct DLL *right;  
};
```



Q. Write a C program to delete a node which contain data as x in the given double linked list.

Ans. del(int x)

```
{ struct DLL *ptr = start;  
  if (start->data ==  $x$ )  
  { start->right = start->right->left;  
    start->left = NULL;  
    free(ptr);  
    ptr = NULL;  
  }
```

```

* if (p != NULL)
{
    exit(1);
}

```

```

else
{
    while (ptr->data != x && ptr != NULL)
    {
        ptr = ptr->right;
        *if (ptr->right == NULL) { (ptr->left)->right = ptr->right;
                                free(ptr); ptr = NULL; }
    }
    else { (ptr->left)->right = ptr->right;
          (ptr->right)->left = ptr->left;
          free(ptr);
          ptr = NULL;
    }
}

```

Q Write a C program to insert a node with data x before a node with data y.

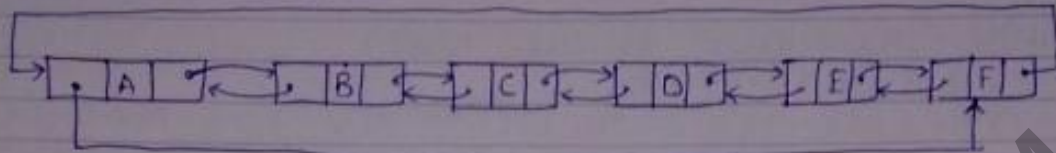
Ans. add (int y, dll * start)

```

{
    dll * ptr = start;
    dll * temp = (dll *) malloc (sizeof (dll));
    while (ptr->data != y && ptr != NULL)
    {
        ptr = ptr->right;
        if (ptr == NULL) { exit(1); }
    }
    if (ptr == start)
    {
        temp->data = x;
        temp->left = NULL;
        start->left = temp;
        temp->right = ptr;
        start = temp;
    }
    else if (ptr == NULL)
    {
        temp->data = x;
        exit(1);
    }
    else
    {
        temp->data = x;
        temp->left = ptr->left;
        temp->right = ptr;
        (ptr->left)->right = temp;
        ptr->left = temp;
    }
}

```


Circular Doubly LL



Q

- (i)
- (ii)

mark all nodes with flag 0, & make the flag as 1 as the node is visited (do this for (i)st LL & then for (ii)nd & check if the flag is 0 or 1, if it is already 1, from that node onwards, count the no. of nodes till NULL) $\rightarrow O(n)$
2n

☆☆ Application of Doubly LL :- Binary Tree

Binary Tree

Q Write a C program to find no. of leaf nodes in the given binary tree

Ans.

```
int totleafnodes (tree * p)
{
    if (p->left == NULL && p->right == NULL)
        return 1;
    else
    {
        totleafnodes (p->left);
        totleafnodes (p->right);
    }
}

void main()
{
    tree * start; // assume it is assigned start root
    int i = 0; // address
    if (start != NULL)
    {
        totleafnodes (start);
    }
}
```

struct tree
{
tree * lc;
int data;
tree * rc;
};

Or

NLN (struct tree *s)

```
{ if (s == NULL) return 0;
  else { if (s->lc == NULL && s->rc == NULL)
    return 1;
    else {
      xL = NLN(s->lc);  $\rightarrow T(n/2)$ 
      xR = NLN(s->rc);  $\rightarrow T(n/2)$ 
      return (xL + xR);  $\rightarrow c$ 
    }
  }
```

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + c$$
$$= 2T\left(\frac{n}{2}\right) + c$$
$$:= \boxed{O(n)}$$

$$\text{or } T(n) = T(a) + T(n-(a+1)) + c$$
$$\text{if } a = 0$$
$$T(n) = \boxed{O(n^2)}$$

Q. Write a C program to find no. of non-leaf nodes in a given binary tree.

```
int i = 0;
totnonleafnodes (tree *p)
{ * if (p->left != NULL || p->right != NULL)
  { i++;
    return;
  }
  else
  { totnonleafnodes (p->left);
    totnonleafnodes (p->right);
  }
}

void main()
{ if (root != NULL)
  { totnonleafnodes (root);
  }
}
```

```
if (p == NULL)
  return;
* if (p->left != NULL || p->right != NULL)
  { return;
  }
```

Or

NLN (tree *s)

```
{ if (s == NULL) return 0;
  else
    { if (s->lc == NULL && s->rc == NULL)
      return 0;
    }
```



```

else
{
    XL = NLN(S->lc);
    XR = NLN(S->rc);
    return(1 + XL + XR);
}
}
}

```

Q Write a C program to find total no. of nodes in the given binary tree.

```

int i = 0;
totnodes(tree *p)
{
    if (p->left == NULL && p->right == NULL) * If (p == NULL) return;
    {
        i++;
        return;
    }
    else
    {
        i++;
        totnodes(p->left);
        totnodes(p->right);
    }
}
}

```

Or

```

NLN(tree *s)
{
    if (s == NULL) return 0;
    else
    {
        if (s->lc == NULL && s->rc == NULL)
            return (1);
        else
        {
            XL = NLN(s->lc);
            XR = NLN(s->rc);
            return (1 + XL + XR);
        }
    }
}
}
}

```

Q Write a C program to find height of a given binary tree.

Ans height (tree * s)

```
{ if (s == NULL) return 0;
```

```
else
```

```
{ if (s->lc == NULL && s->rc == NULL)
```

```
return 0;
```

```
else
```

```
{ xL = height(s->lc);
```

```
xR = height(s->rc);
```

```
return (1 + max(xL, xR));
```

```
}
```

```
}
```

Or
height (tree * p)
{ n = no. of nodes

Q Write a C program to check given binary tree is strict binary tree or not.

bool flag = true;

strictOrNot (tree * p)

```
{ if (p->left == NULL && p->right == NULL)
```

```
{ return,
```

```
}
```

```
else if ((p->left == NULL && p->right != NULL) ||
```

```
(p->left != NULL && p->right == NULL))
```

```
{ flag = false;
```

```
return,
```

```
}
```

```
else
```

```
{ strictOrNot (p->left);
```

```
strictOrNot (p->right);
```

```
}
```

```
}
```

Or

strictBT (tree * s)

```
{ if (s == NULL) return 1;
```

```
else
```

```
{ if (s->lc == NULL && s->rc == NULL)
```

```
return 1;
```

```
else
```



```

if (s->lc != NULL && s->rc != NULL)
    return (strictBT(s->lc) && strictBT(s->rc));
else
    return 0;
}

```

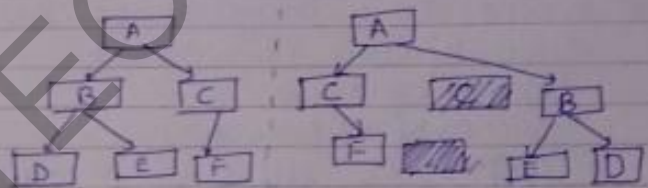
Q Consider the following C program:-

```

Tree (struct BtNode *s)
{
    if (s != NULL)
    {
        t = (struct BtNode *) malloc (sizeof (struct BtNode));
        t->data = s->data;
        t->lc = s->rc; tree (s->rc);
        t->rc = s->lc; tree (s->lc);
        return (t);
    }
}

```

Mirror Image



Q Write a C program to find min. element in the given binary search tree.

Ans. `int min = root->data;`

`findmin (tree *p)`

`{ if (p->left == NULL) return (p->right->data);`

`return (p->data);`

`if (p->data <`

`findmin(p->left);`

`}`

`or`
`findmin (tree *s)`

`{ if (s == NULL) return;`

`else`

`{ while (s->lc != NULL)`

`s = s->lc;`

`return (s->data);`

`}`
`}`

Q. A data structure comprises of nodes, each of which has exactly 2 pointers to other nodes with no Null pointers, the following C program is to be used to count the no. of nodes in given binary tree. It uses a mark field assumed to be '0' initially for all nodes. Find the missing statement in the code:-

→ struct test

```
{ int info, mark;
  struct test *p, *q;
};
```

```
int nodecount(struct test *a)
```

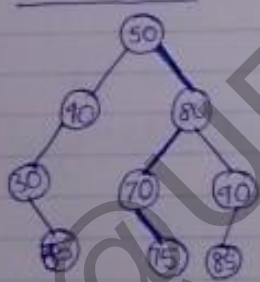
```
{ if (a->mark)
  return (0);
```

```
else
  { return (nodecount(a->p) + nodecount(a->q) + 1);
  }
```

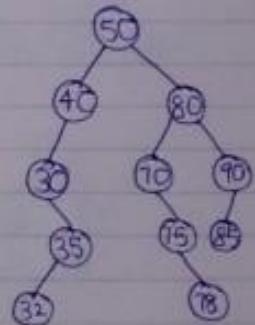
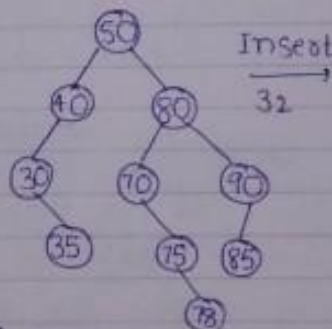
```
→ a->mark = 1; (my answer)
```

Binary Tree

Insertion



Insert - 78



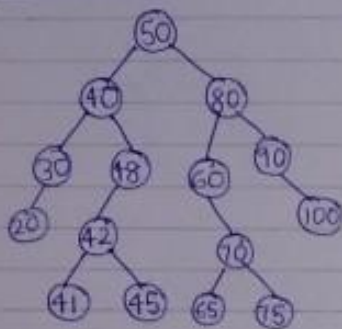
Finding the position to insert:-

- Best Case:- $O(1)$ (root itself)
- Worst Case:- $O(n)$
- Avg Case:- $O(\lg n)$

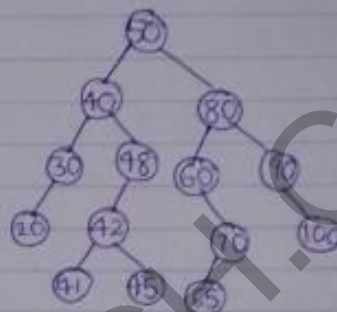
Algo

- ① Find the place of $x \rightarrow O(n)$
- ② Insert element $x \rightarrow O(1)$
 $O(n)$

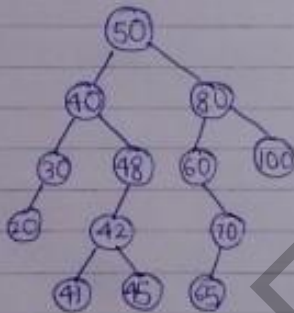
Deletion



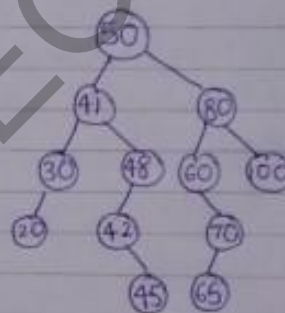
Delete 75



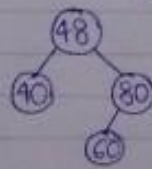
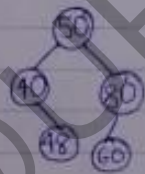
Delete 100



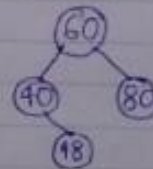
Delete 40



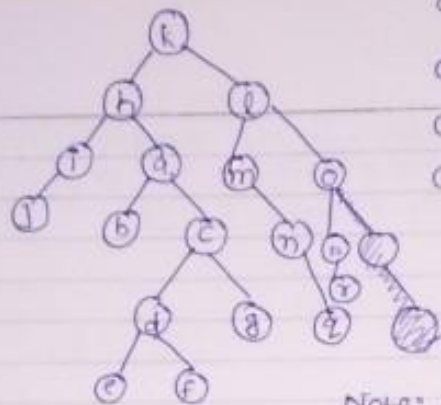
delete 50



or



either choose the Left Subtree largest child or Right Subtree smallest child.



delete(a) = inorder predecessor or
inorder successor $\rightarrow b(x)$

delete(c) = e f (or) g

delete(d) = e (or) f

delete(o) = connect l & p

delete(q) = no-need to do any

$$n + O(1) = \boxed{O(n)}$$

finding for
place adjustment

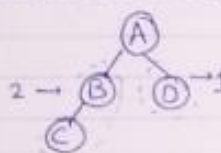
Note: If we delete any node from BST
replacement will be done by inorder
successor or inorder predecessor.

AVL Tree

(Adelson, Valski, Landies)

Or Balanced Binary Search tree [Size $\rightarrow O(\log n)$]

Balanced Factor = $H(LST) - H(RST)$ or $H(RST) - H(LST)$



Balanced Factor (of A) = $2 - 1 = 1$

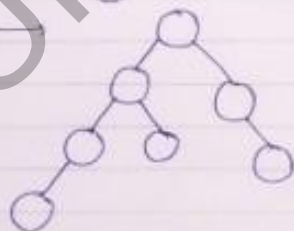
Note: A BST in which every node's Balance factor is $-1, 0$ or 1 (max. diff) is called AVL tree.

Height = 2 $\rightarrow$



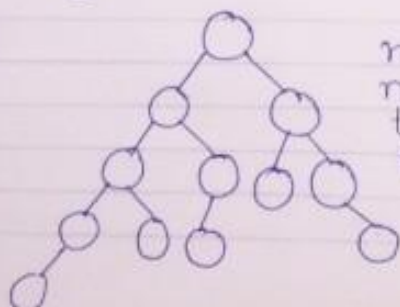
min no. of
nodes for a tree
to be AVL of
height 2 = 4

Height = 3 $\rightarrow$



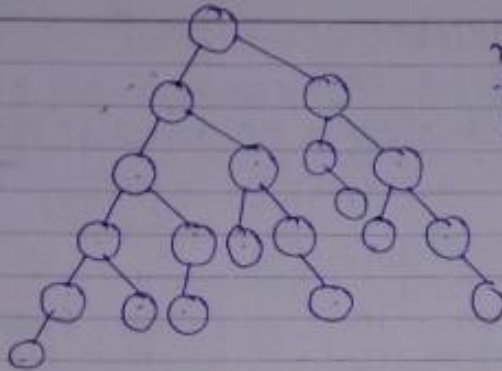
min no. of
nodes for a tree
to be AVL of
height 3 = 7

Height = 4 $\rightarrow$



min no. of
nodes for a
tree to be AVL of
height 4 = 12

height = 5



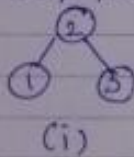
min. no. of nodes
for a tree to be AVL
of height 5 = 20

Min. no. of nodes in a height H AVL tree: MNN

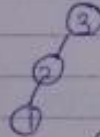
$$MNN(H) = MNN(H-1) + MNN(H-2) + 1$$

$$MNN(H-1) + MNN(H-2) + 1$$

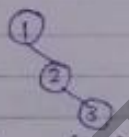
Q. $n = 3(1, 2, 3) \rightarrow$ BST



(i)



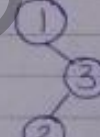
(ii)



(iii)



(iv)



(v)

AVL trees: - (i)

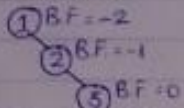
BST: - (i), (ii), (iii), (iv), (v)

(ii), (iii), (iv), (v) are converted to (i) using rotations, & this rotation should take constant time.

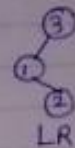
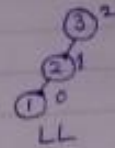
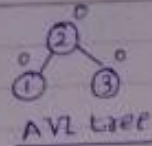
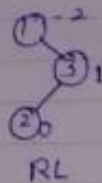
Date

21.08.12 AVL Trees

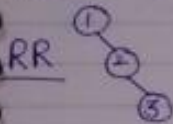
$n=3 \Rightarrow (1,2,3)$



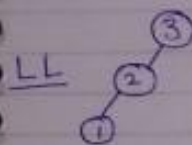
In this case ① has problem from R (because of ②) & ② has problem from its right, RR problem.



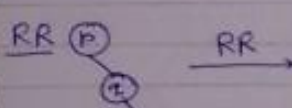
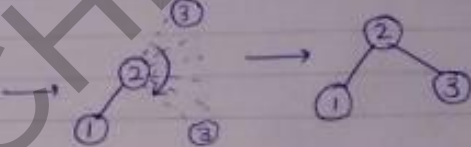
Rotations :-



1. Keep the nodes as it is which has B.F. = 0, 1, -1 & rotate the node which has B.F. $\neq 0, 1, -1$ to the anticlockwise (one left rotation)

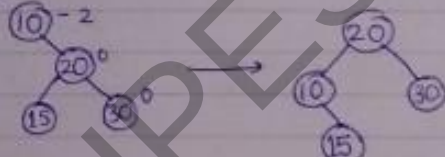


1. Keep the nodes as it is which has B.F. = 0, 1, -1 & rotate the node which has B.F. $\neq 0, 1, -1$ clockwise (one right rotation)

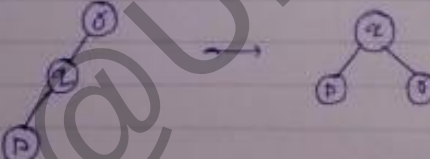


$q \rightarrow lc = p$
 $p \rightarrow rc = NULL$

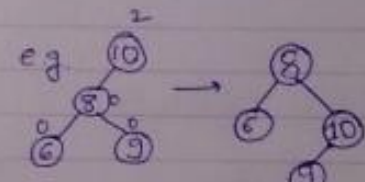
e.g.



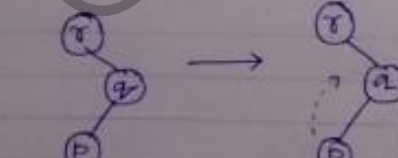
LL



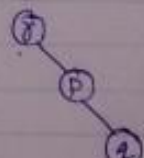
$q \rightarrow rc = r$
 $r \rightarrow lc = NULL$



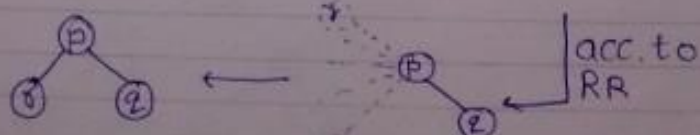
RL

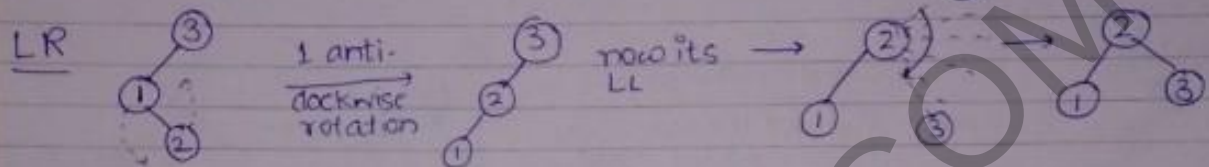
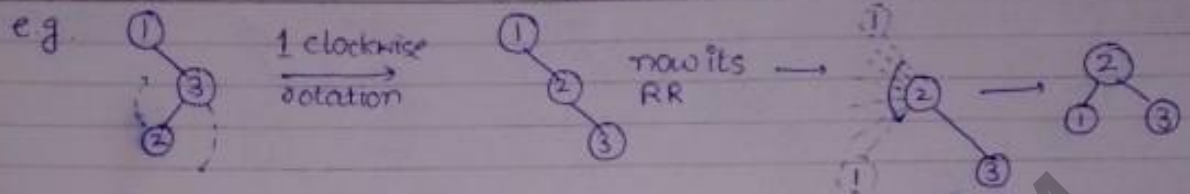


1. do one clockwise rotation

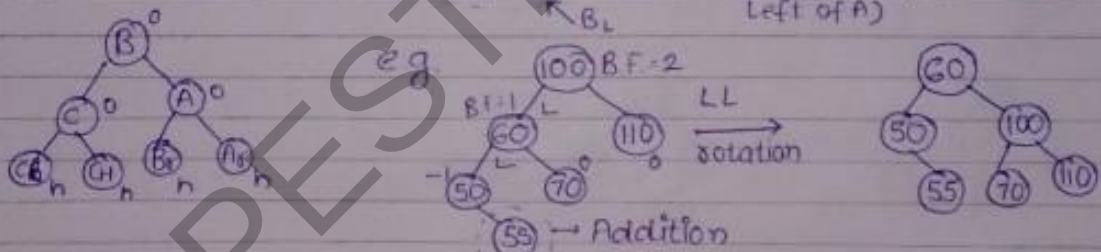
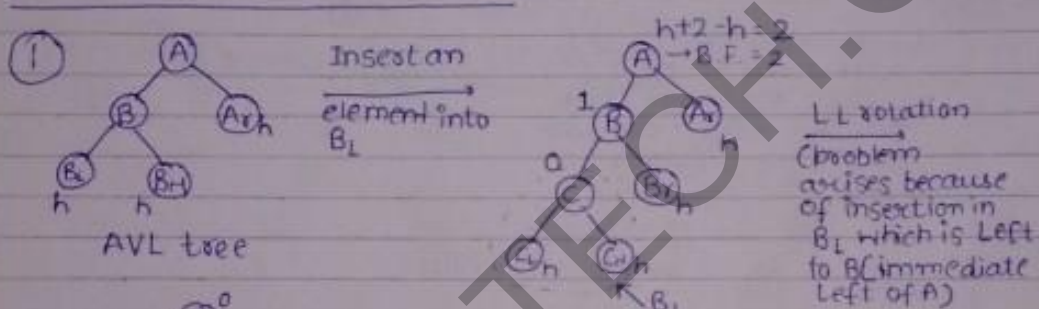


now this has RR problem.

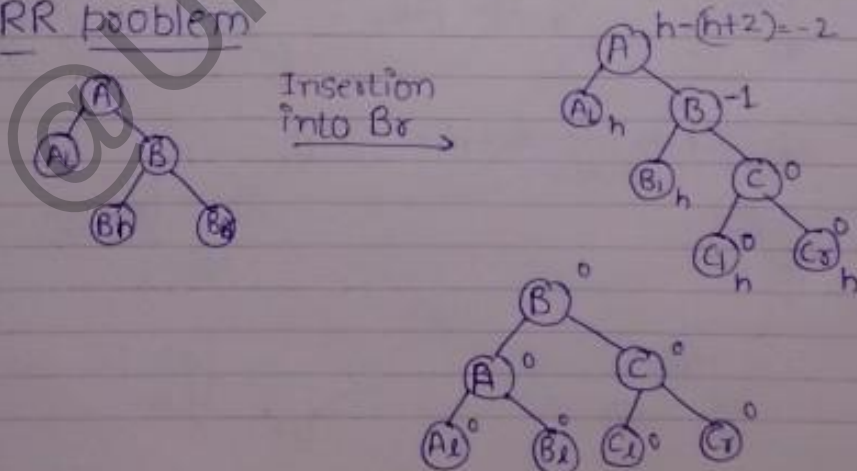




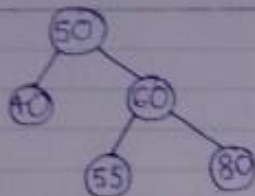
Insertion into AVL tree:-



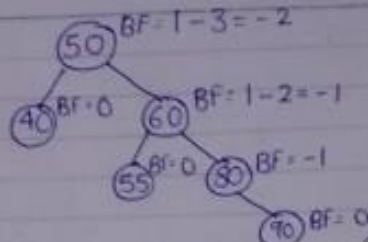
② RR problem



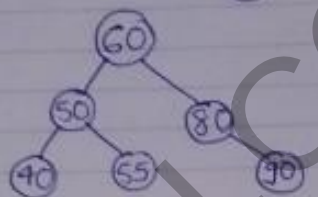
e.g



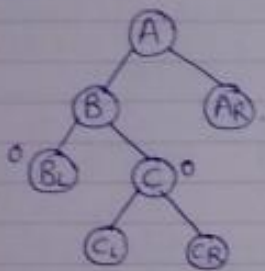
Add
90



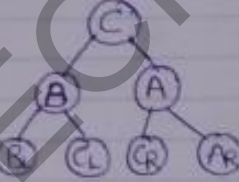
RR



LR

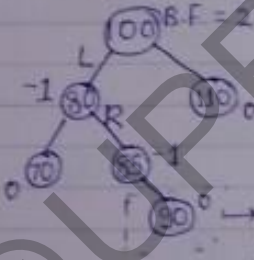


LR



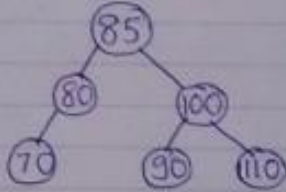
★ In LR problem 2 rotations are compulsory.
★ After doing one rotation we will get LL. After getting LL problem, apply LL rotation.

e.g

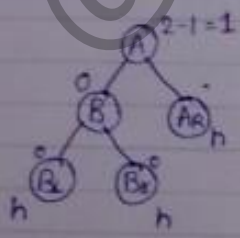


Insertion of 90.

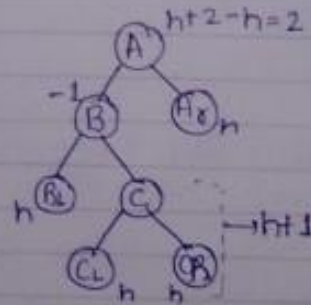
LR



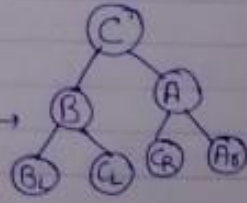
Sis's explanation:-



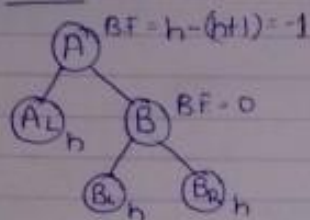
Insert element in Br



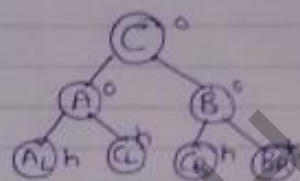
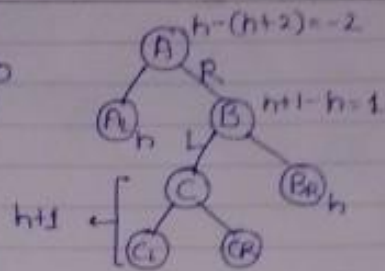
LR



RL

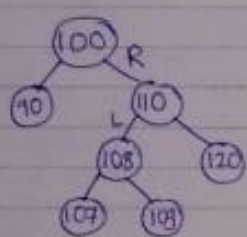


Insert into
 B_L

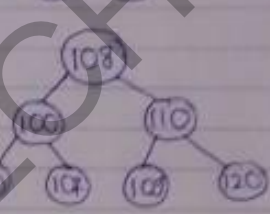


RL rotation

e.g.

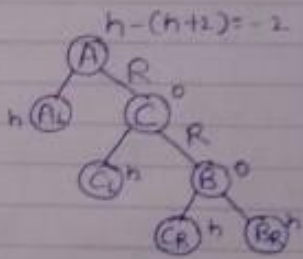
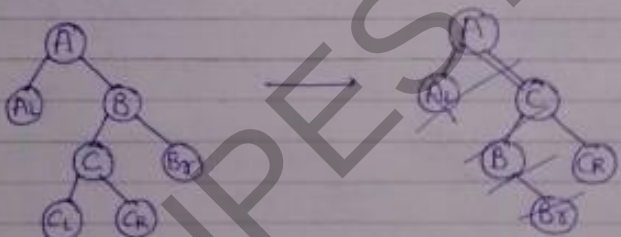


RL
problem

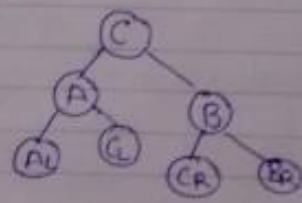


* In RL rotation
2 reverse rotations
are compulsory
* After 1 rotation
we will get RR
problem.
* Apply 1 more
RR rotation to
get AVL tree.

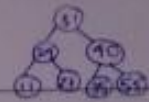
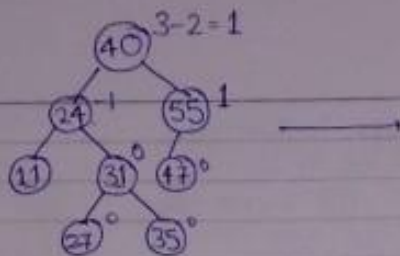
Or



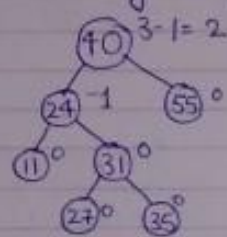
RR



Q Consider the following AVL tree:-



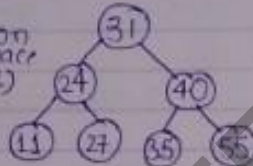
After deleting 47, show resultant AVL tree -



★ To delete an element,
Search that element,
Searching complexity :- $O(n)$ [BST] & not AVL
& then perform
the deletion algorithm.

$O(\log n)$ [AVL & BST]

LR (Deletion
takes place
Left from RL)
rotation



Struct AVL

{ int data;

int height;

struct AVL* lc, *rc;

};

Complexity :-

① Finding place of deletion
→ $\log n$

② Deletion of that element
→ Constant

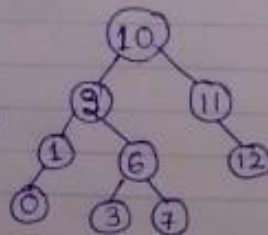
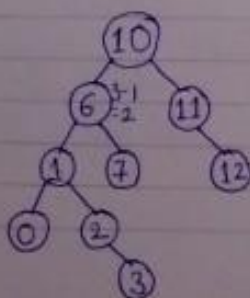
③ Traversing the complete
path from site of deletion
to the root to check the
balancing factors = $\log n$

$$2 \log n = O(\log n)$$

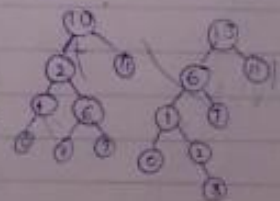
Q. What will be the resulting AVL tree if we insert following
elements into AVL tree:-

10, 6, 11, 12, 1, 7, 0, 2, 3

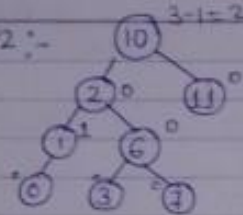
Addition → $O(\log n)$



AVL Tree



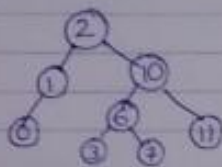
delete 12 :- $3-1=2$



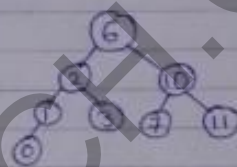
Now problem is because of left subtree of 10, L, now we are at 2, we can 2 has B F = 0, we can go both left & right, going left results in LL
" " " " LR

What will be the resulting AVL tree if we delete 12.

LL —



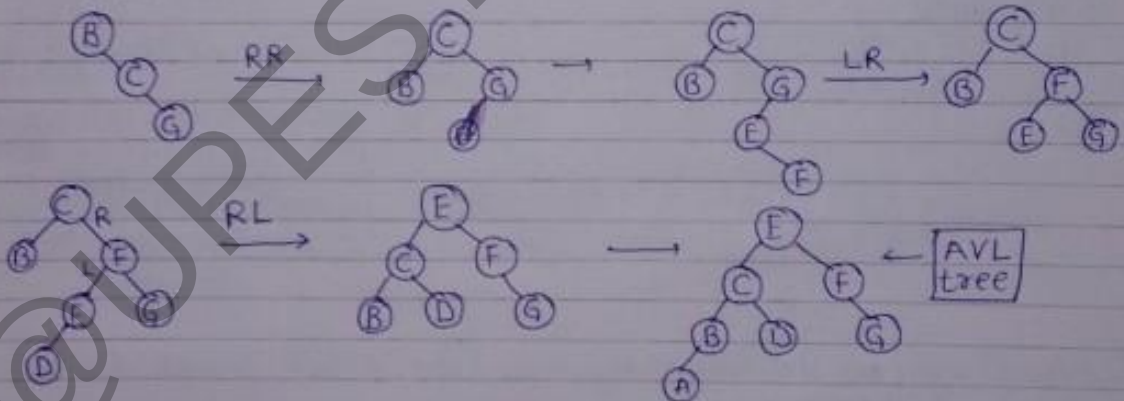
1 rotation

 $LR \rightarrow$ 

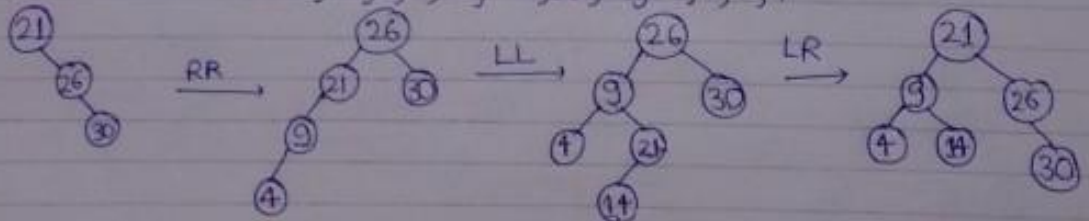
2 notations

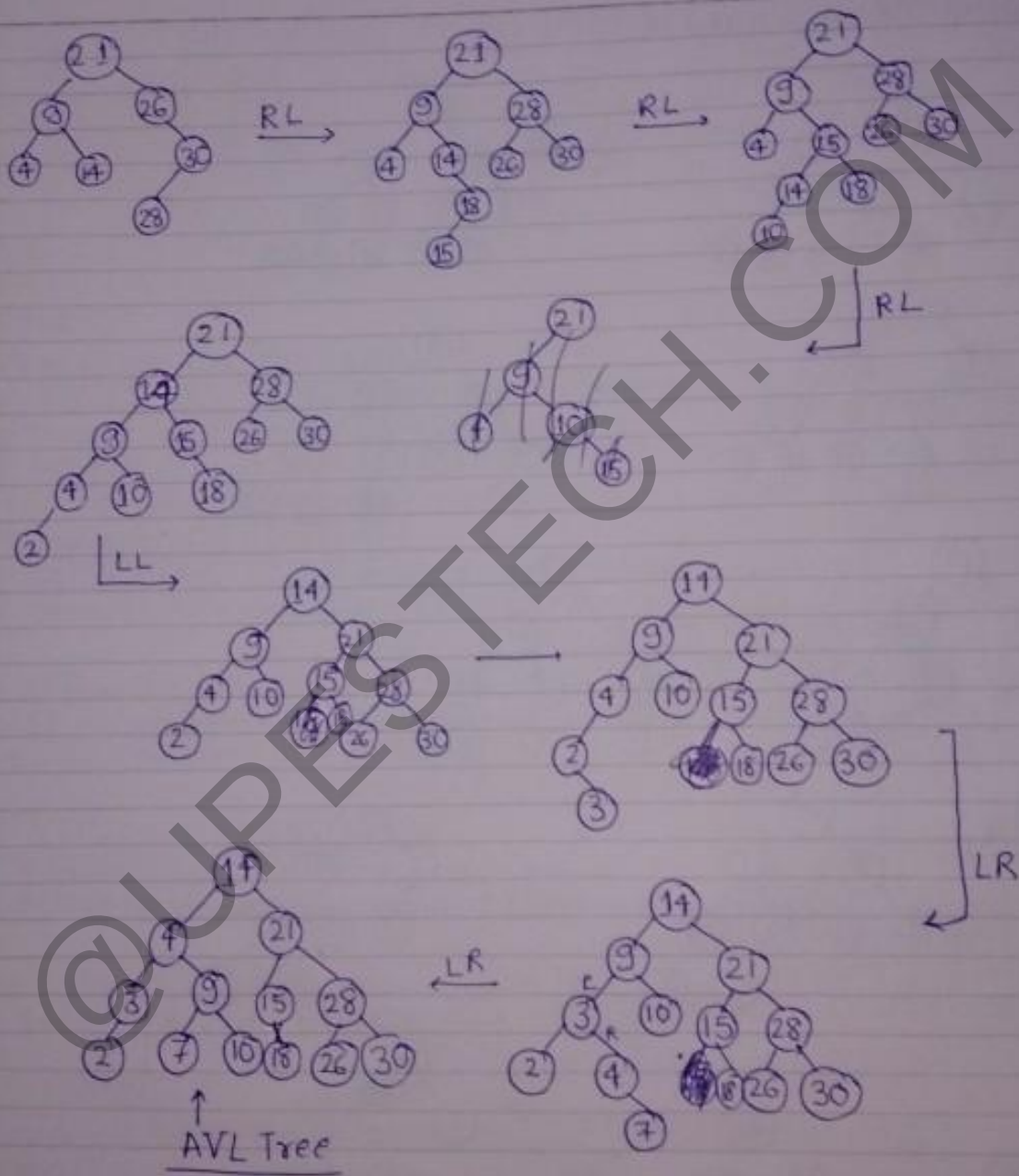
We could apply any of these. (either LL or LR.)

Q. Construct AVL trees from following keys
B, C, G, E, F, D, A

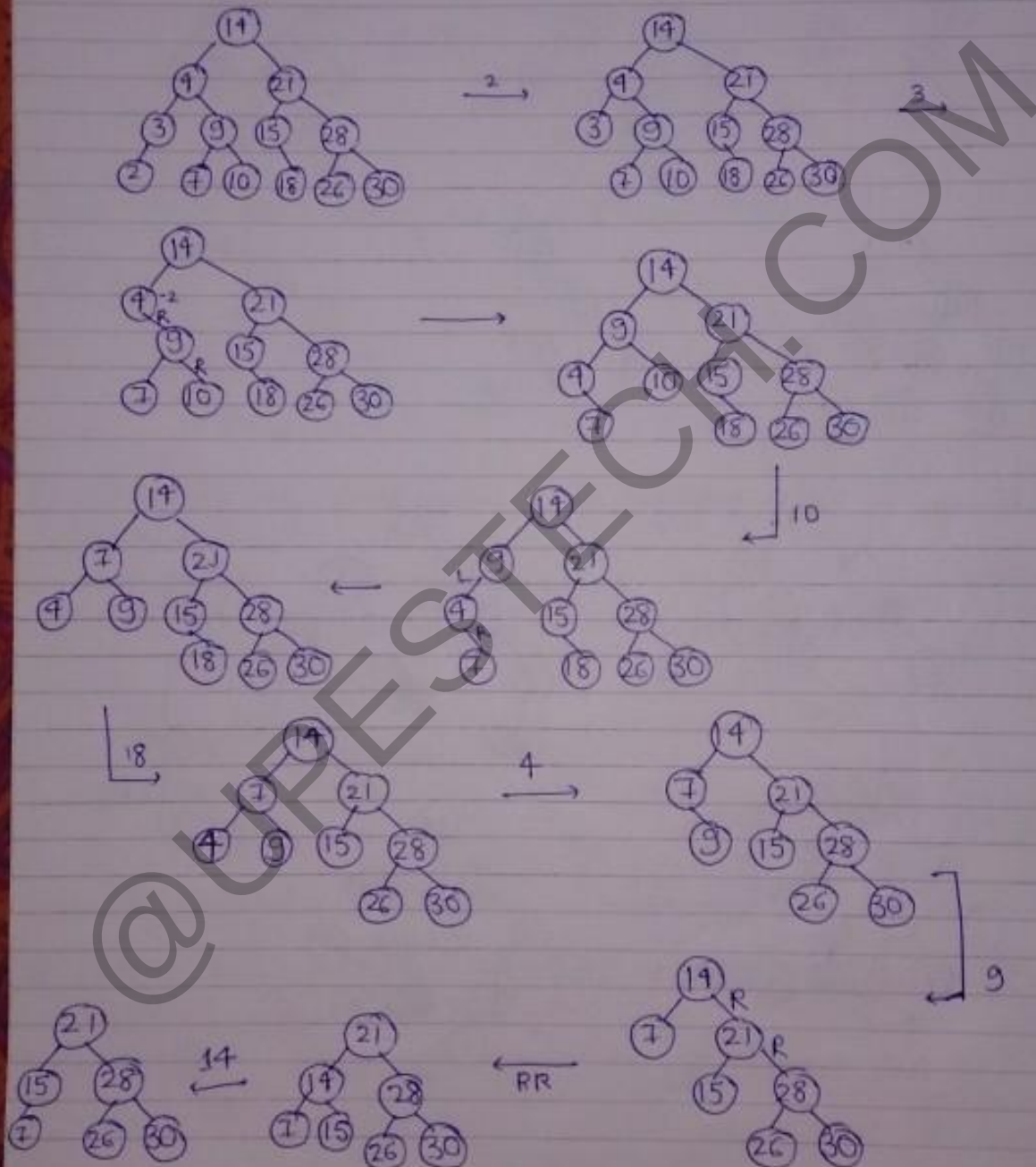


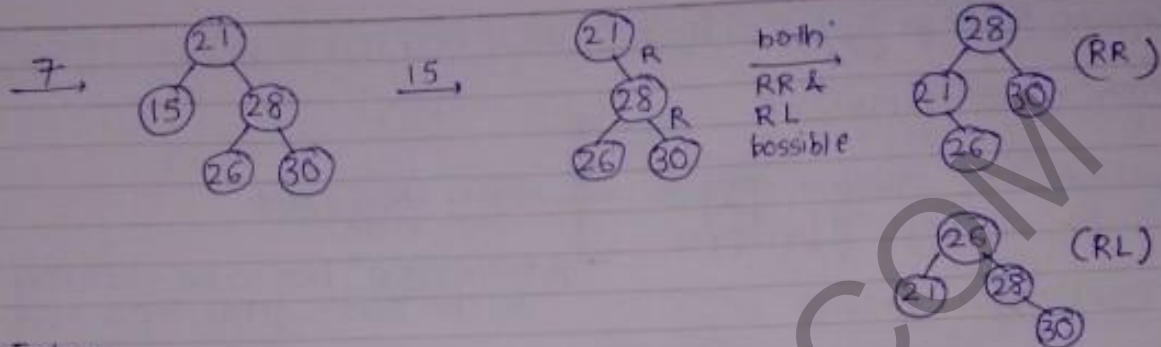
Q. Create AVL- 21, 26, 30, 9, 4, 14, 28, 18, 15, 10, 2, 3, 7





Q. Delete 2, 3, 10, 18, 4, 9, 14, 7, 15





Note:-

Algorithm for insertion

1. Find the place of the element $\rightarrow O(\log n)$
2. Insert that element $\rightarrow O(1)$
3. Traverse the same path to find BF $\rightarrow O(\log n)$
4. If not balanced at some node, perform rotation $\rightarrow O(1)$

Total Complexity: $O(\log n)$

Algorithm for deletion

1. Find the place of the element to be deleted $\rightarrow O(\log n)$
2. Deletion of element $\rightarrow O(\log n)$ [when there are 2 children of a node to be deleted]
 $\rightarrow O(1)$ [when children are 0 or 1.]
3. Check the balancing factors $\rightarrow O(\log n)$ by traversing
4. If not balanced at some node, perform rotation $\rightarrow O(1)$

Total Complexity: $O(\log n)$

Multiway-Search Tree

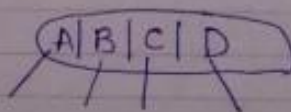
1. Search Tree (Ordered Tree)
2. At every node minimum 2 children allowed (order of multiway search tree m).
3. Height of Multiway search tree is $O(\log_m n)$. [Balanced M-way search tree.]
4. Deletion & Insertion will take $O(\log_m n)$.

B-Tree:-

Order of B-Tree = m (assume) [means, each node can have max. m children]

$\therefore$ maximum $(m-1)$ keys

maximum m -children



min. $\lceil \frac{m}{2} \rceil$ children

min. $\lceil \frac{m}{2} \rceil - 1$ keys.

Degree = 5 | Keys: 5, 10, 12, 13, 14, 15, 2, 3, 4, 20, 18, 13, 17, 16, 15, 25, 23, 24, 22, 21, 30, 28, 29

max. no. of children = 5

" " " keys = 4

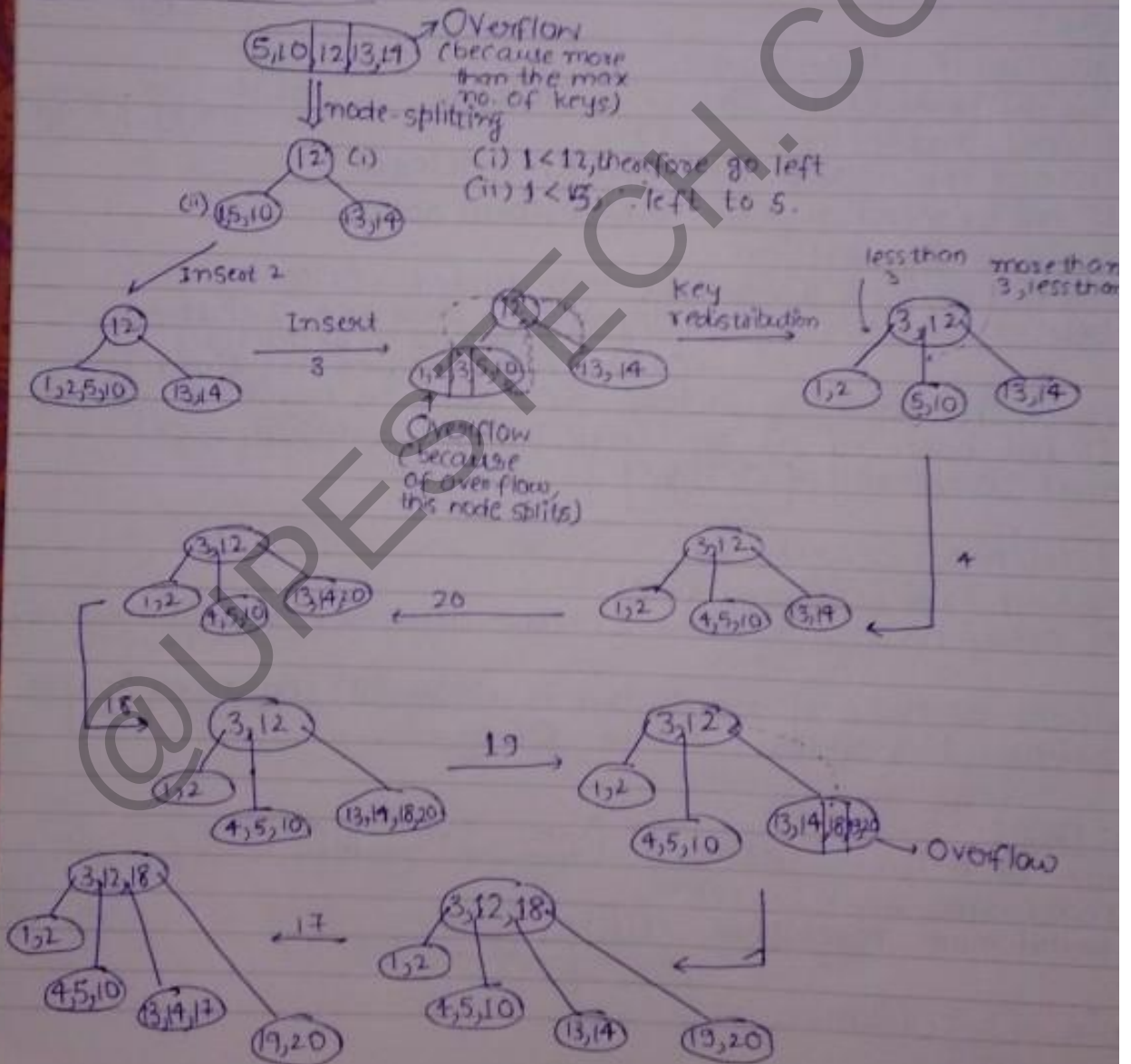
min no. of children = $\lceil \frac{5}{2} \rceil = 3$

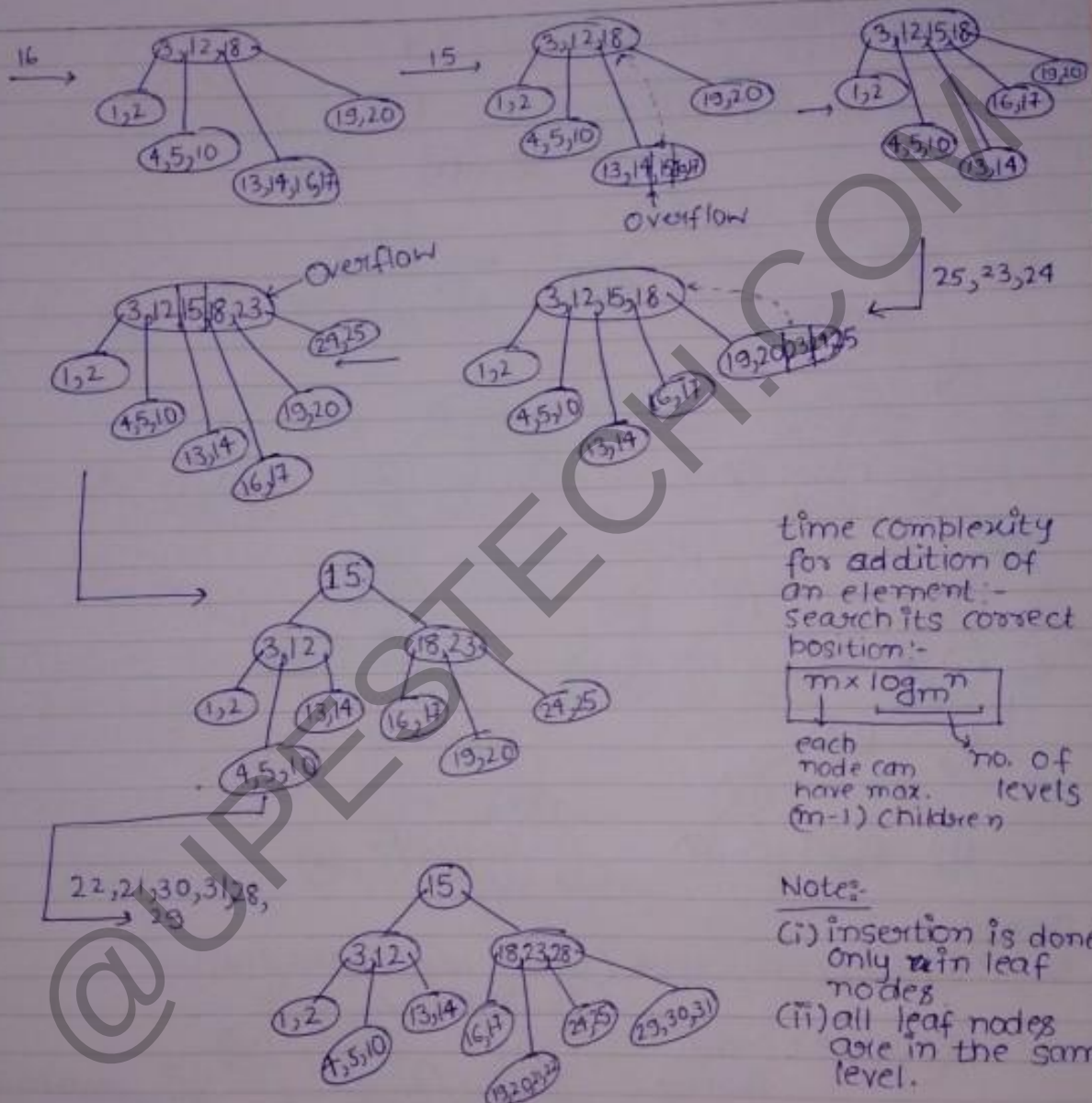
min. no. of keys = $\lceil \frac{5}{2} \rceil - 1 = 3 - 1 = 2$

Other than root node

A min. condⁿ has to be satisfied by the nodes other than root

Insertion in B-Trees





time complexity for addition of an element - search its correct position:-

$$\boxed{m \times \log_m n}$$

each node can have max. $(m-1)$ children

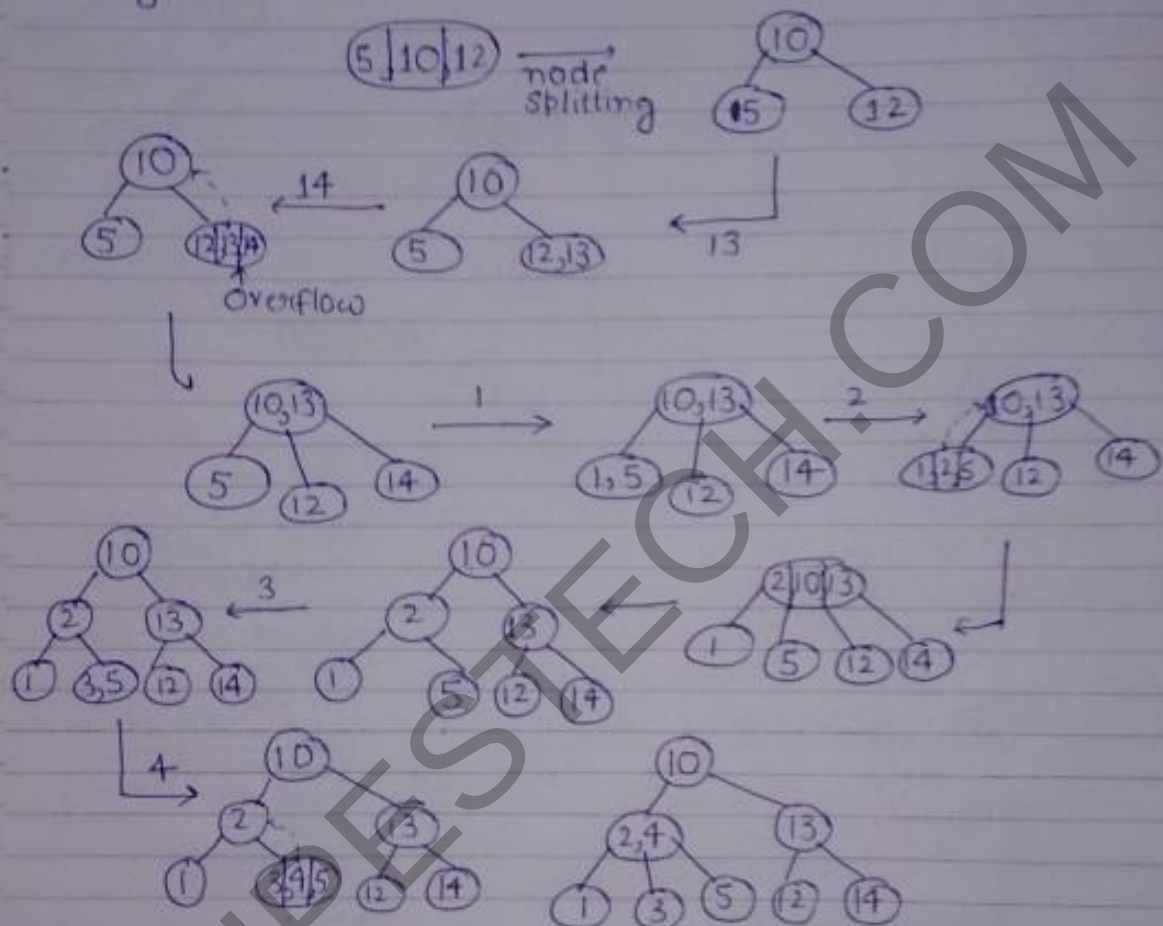
no. of levels

Notes:-

- (i) insertion is done only in leaf nodes
- (ii) all leaf nodes are in the same level.

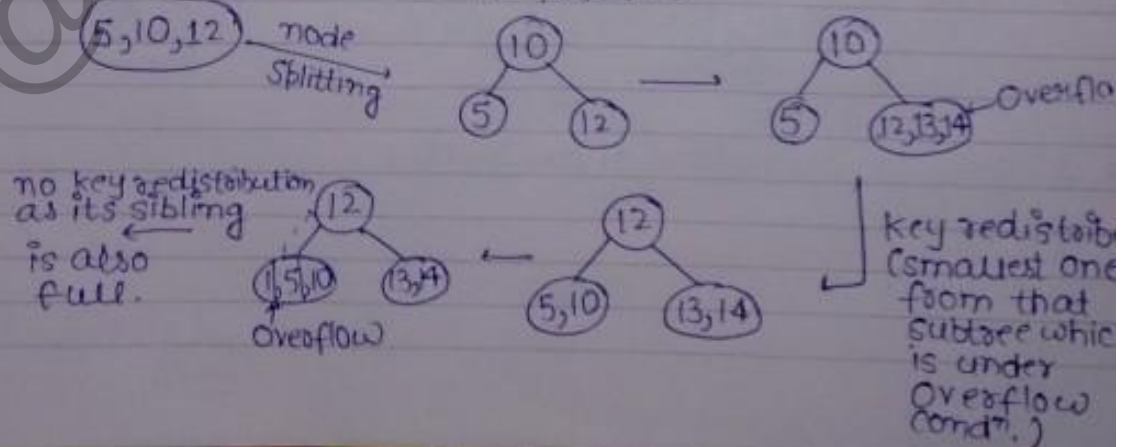
★ ★ A 2-3 tree means max. 3 child

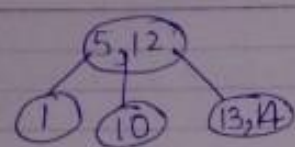
Q. degree = 3 | keys: 5, 10, 12, 13, 14, 1, 2, 3, 4



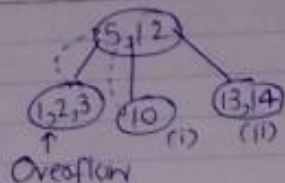
Q. Degree = 3

Keys: 5, 10, 12, 3, 14, 1, 2, 3, 4 (using key redistribution if possible)

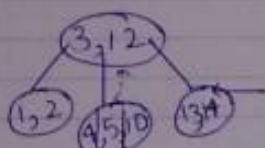




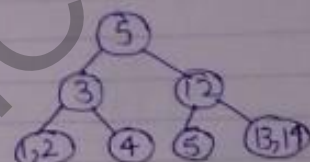
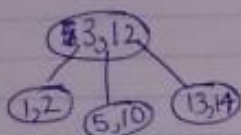
1, 2



if both the siblings (left & right) have space, carry on redistribution (left or right) for key redistribution, ask only the immediate siblings so ask for key redistribution with (1) only.



no key redistribution



Note:- While constructing B-Tree for the given trees, key-redistribution is always better than node splitting, because we are not wasting space & height of B-Tree also decreases, so time complexity also decreases

Q. Consider the following B-Tree:-

2-3-4 Tree (A B-Tree with min. degree of 2, ∴ max min. children = 2
min. keys = 1
max. children = 4
max. keys = 3)

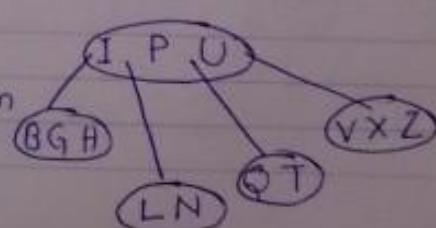
in which each data item is a letter



what is the result of inserting G in the above tree:-



key redistribution



Q. Consider a B-Tree with order = 32

(i) find max. no. of nodes present in a B-Tree with height

3.

(ii) find min. no. of nodes present in a B-Tree with height 3.

Ans. (i) height = $\log_{32} n$

$$3 \leq \log_{32} n$$

$$3+1 = \log_{32} n$$

$$4 = \log_{32} n$$

$$32^4 = n$$

$$32^4 = n$$

max. children = 32

max. keys = 31

min. children = 16

min. keys = 15

for max.

Level	nodes	keys	children
1	1	31	32
2	32	32×31	32×32
3	32×32	$32 \times 32 \times 31$	$32 \times 32 \times 32$
4	$32 \times 32 \times 32$	$32 \times 32 \times 32 \times 31$	$32 \times 32 \times 32 \times 32$

$$\text{Total nodes} = 32^0 + 32^1 + 32^2 + 32^3$$

$$= \frac{32^4 - 1}{32 - 1}$$

$$= 32^3$$

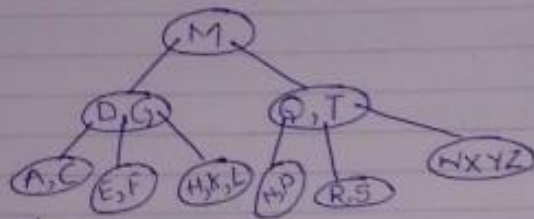
max. no. of nodes in a height h , order m B-Tree:-

$$= m^0 + m^1 + \dots + m^h$$

$$= \frac{1(m^{h+1} - 1)}{m - 1} = \frac{m^{h+1} - 1}{m - 1}$$

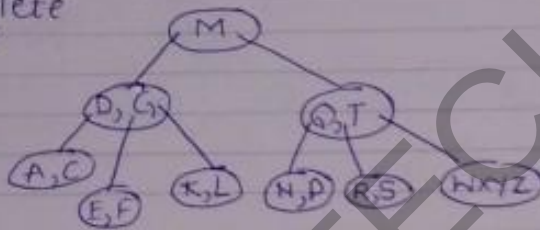
$$\text{max. keys} = (m-1) \left[\frac{m^{h+1} - 1}{m - 1} \right] = \frac{(m-1)(m^{h+1} - 1)}{(m-1)} = (m^{h+1} - 1)$$

Q. Consider the following B-Tree

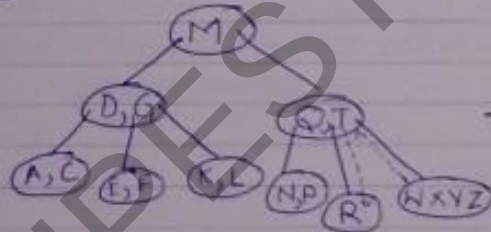


max. no. of keys = 4
 order = 5
 min no. of children = $\lceil \frac{5}{2} \rceil = 3$
 min no. of keys = $3 - 1 = 2$

Delete
H



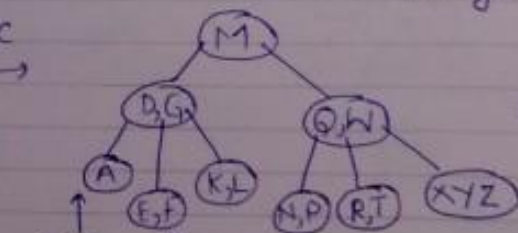
Delete G



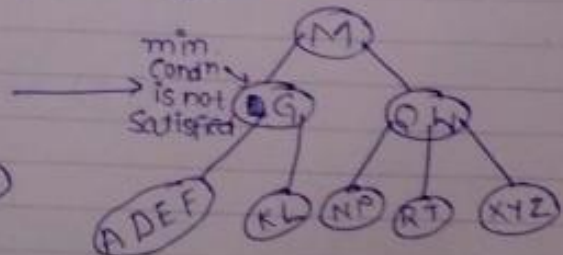
* If the element H is present in leaf node, delete that element & check for the min. condn. If it is satisfied, then do nothing, otherwise

min. condn not satisfied
 ask left sibling → can't give
 ask right → can "]
 [if min. condn is not satisfied, distribute the keys from immediate sibling (if possible)].

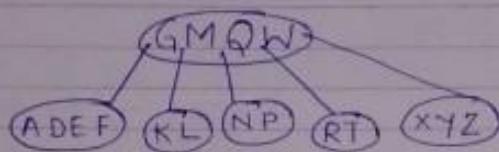
Delete C



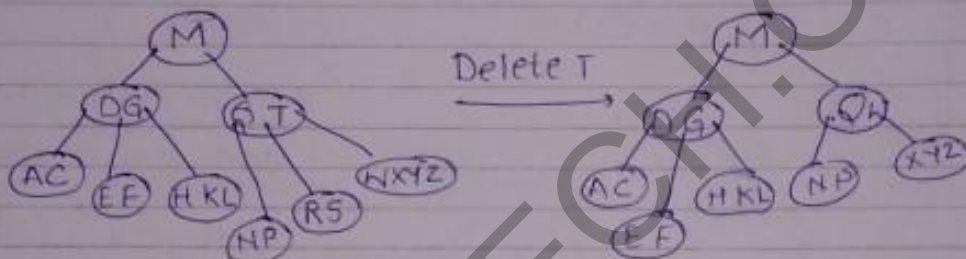
min. condn is not satisfied
 sibling can't help, so node merging



G asks the sibling, but it can't help, ∴ node merging

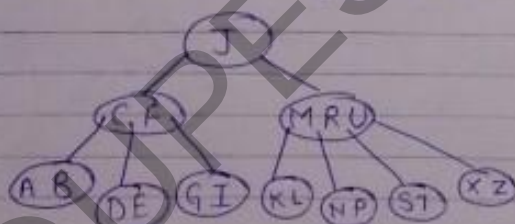


Q. Consider the following B-Tree.



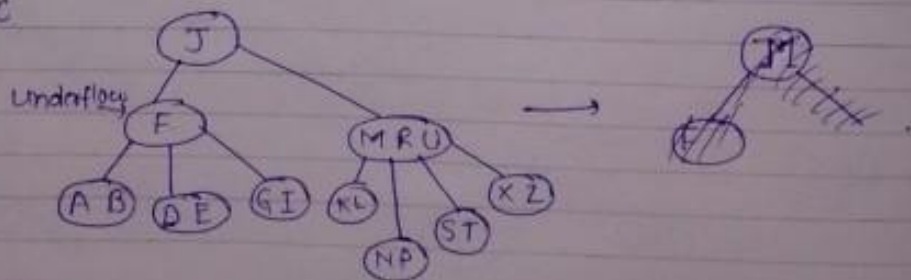
★ If we delete any element which is present in non-leaf node, replacement will be done by children, then if not by children, then it is done by sibling, still not satisfied merge with parent.

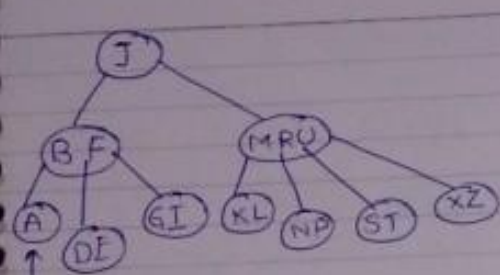
Q.



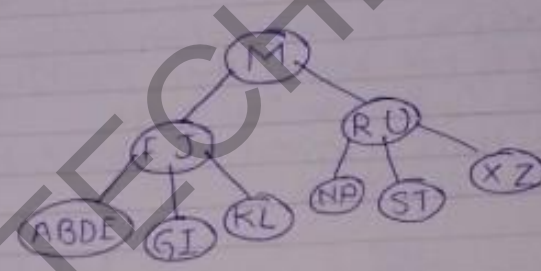
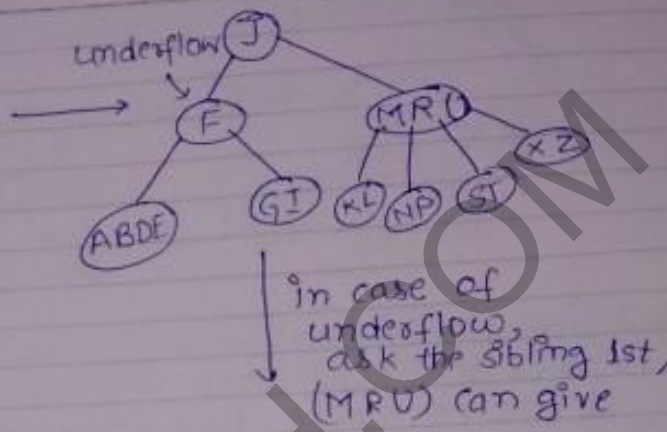
min. keys = 2
min. children = 3
max. keys = 4
max. children = 5

Delete C

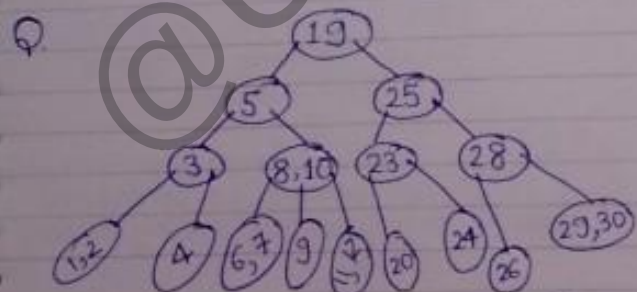




underflow
 know A will ask its parent, sibling if not satisfied by its sibling then merge with parent
 [in case of underflow, ask the parent, in case of deletion, replace blindly with children & then perform other opⁿ]

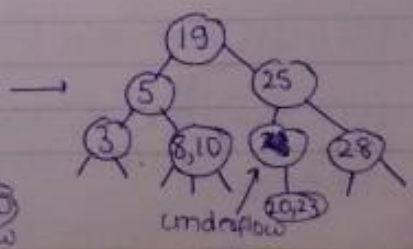
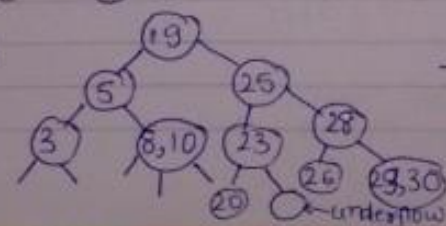


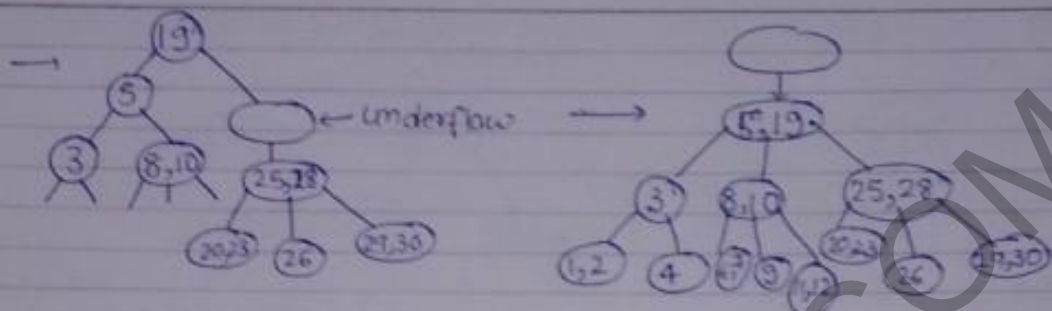
★ In case of element deleted from non-leaf node, replace the element (or take the element) from children & then perform opⁿ of underflow (if there is some underflow in children).
 In case of underflow, ask help from siblings, if not then merge with parents.



min - keys = 1
 min - children = 2
 max - children = 4
 max. keys = 3

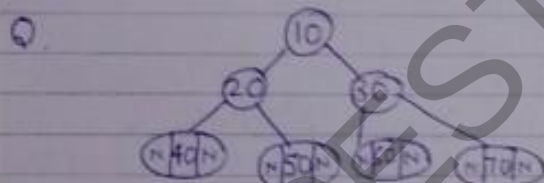
Delete 24



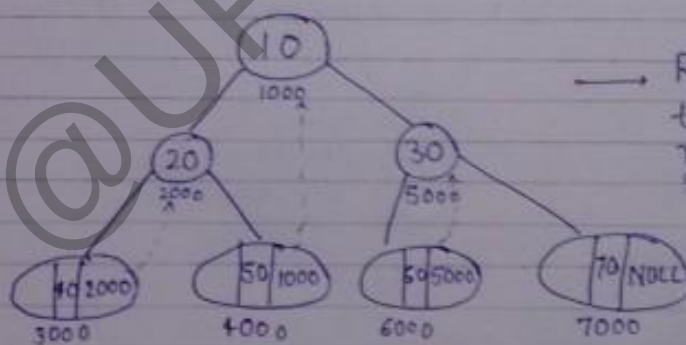


Deletion Algorithm:-

1. Element is in leaf node
delete element & check for underflow
if underflow happened then borrow keys from siblings. If sibling is not ready then combine with the parent.
2. Element is in non-leaf node
delete that element & replace by inorder predecessor or inorder successor.



1. In Binary Tree, all leaf nodes left pointer & right pointer are null.



→ Right In threaded binary tree (right pt. of leaf node contain inorder successor address)

↓
To find inorder no. need of recursive procedure (no stack req. using threaded BT.)

→ Left-In Threaded Binary Tree

↓
Left pointer of leaf nodes contain inorder successor address.

Q In order 32 B-Tree with height 3, find min. keys & min. nodes

level	nodes	keys	children [min]
1	1	1	2
2	2	15×2	2×16
3	$32(2 \times 16)$	32×15	$2 \times 16 \times 16$
4	$2 \times 16 \times 16$	$2 \times 16 \times 16 \times 15$	0

Order m B-Tree with height h contain min. no. of nodes:-

$$1 + 2 \times \left(\frac{m}{2}\right)^0 + 2 \times \left(\frac{m}{2}\right)^1 + 2 \times \left(\frac{m}{2}\right)^2 + \dots + 2 \times \left(\frac{m}{2}\right)^{h-1}$$

$$= 1 + 2 \times \left\{ \frac{\left(\left(\frac{m}{2}\right)^h - 1\right)}{\left(\frac{m}{2} - 1\right)} \right\} = 1 + 2 \times \left\{ \frac{\left(\frac{m}{2}\right)^h - 1}{\left(\frac{m}{2} - 1\right)} \right\}$$

Stack

The side is open another side is closed.

LIFO OR FILO

Top is variable which contain position of top most elem.

ADT of Stack (what opⁿ perform on data structure)

1. push (Stack, e) = Stack operation

2. pop (Stack) = element of top

3. Is empty (Stack) = Boolean

4. Full (Stack) = Boolean

Implementing Stack using array

```
# int a[5] = { - , - , - }
```

```
  a[10] = 0;
```

```
  pf (a[10]); [ a[10] is not allocated to you it give  
               garbage ]
```

Segmentation error = Mem is not available.

operation (push)



```
push(a, N, TOP)
```

2: Name of stack or array

top - of stack

N - size of stack, array

DS

top = 23 (insertion)

```
# push(S, N, TOP, X)
{
```

```
    if (TOP == N-1)
```

```
    {
        printf("Stack is overflow");
        exit(1) ⇒ abnormal Termination
    }
```

```
    else
```

```
    {
        TOP++;
        S[TOP] = X
    }
```

[S[TOP+1] = X [ready to data loss]]
[S[1+TOP] = X (✓)]

deletion TOP
--TOP = 2
--TOP = 1
--TOP = 0
--TOP = -1

⇒ O(1)

```
pop(S, n, TOP)
```

```
{
```

```
    if (TOP == -1)
```

```
    {
        printf("Stack is underflow"); ⇒ O(1)
        exit(1)
    }
```

```
    else
```

```
    {
        int y = S[TOP]
        printf("%d", y);
        TOP--;
    }
```

```
}
```


5	4
4	3
3	2
2	1
1	

$$x = y$$

Let S be a empty stack, storing from empty stack n natural no are pushed into stack. when they then perform 'n' pop opn. Assume push and pop opn will take 'x' second each and y second elapsed b/w end of such stack and before the start of next opn.

⇒ Stack life of element 'm' defined as the time elapsed from push(m) to before the pop(m)

find average stack life of an element in stack

- a) $n \times (n+1)y$ b) $3y + 2x$ c) $(n+1)y - x$ d) None

push					
pop					
	c	b	a	d	

lib time of d = 4 (x=2, y=4)

$$1) \quad 1) \quad c = 1, 2, 4, 4, 2, 4 = 16$$

$$2) \quad 2) \quad b = 1, 2, 4, 2, 4, 4, 2, 4 = 28$$

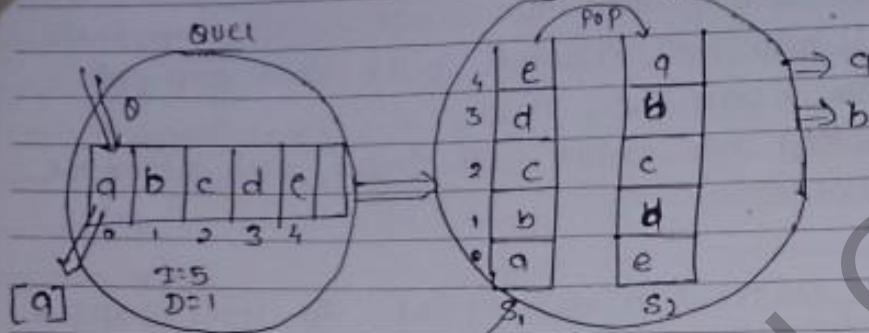
$$3) \quad 3) \quad a = 1, 2, 4, 2, 4, 2, 4, 4, 2, 4 = 40$$

$$4 + 16 + 28 + 40 = 88$$

$$\frac{4 + 16 + 28 + 40}{4}$$

4

Q1) write a c program to implement queue using stack



1. 5 - push in S1

2. 5 - pop from S1

3. 5 - push into S2

4. 1 - pop from S2

⇒ a (give 'a' as deleted element)

1. pop from S2 ⇒ a

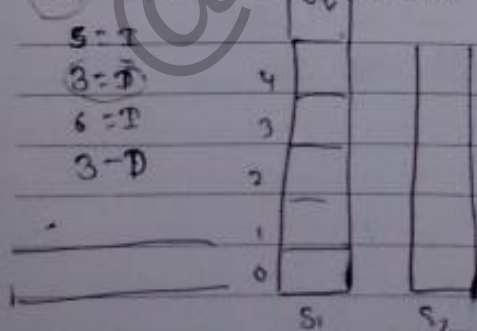
2. pop from S2 ⇒ b

3. pop from S2 ⇒ c

4. pop from S2 ⇒ d

5. pop from S2 ⇒ e

Q2) queue using stack



5 = push - S1 ✓

5 = pop - S1 ✓

5 = pop - S2 ✓

3 = pop - S2 ✓

6 = push - S1 ✓

2 = pop - S2 ✓

6 = pop - S1 ✓

6 = push - S2 ✓

2 = pop - S2 ✓

pop: 5 + 3 + 2 + 4 + 3 = 15 pop

22 push

17 pop

program

Dequeue()

```

{
  if (s2 is empty)
  {
    if (s1 is empty)
      printf("underflow")
    else
    {
      while (s1 is not empty)
      {
        x = pop(s1);
        push(s2, x);
      }
    }
  }
}

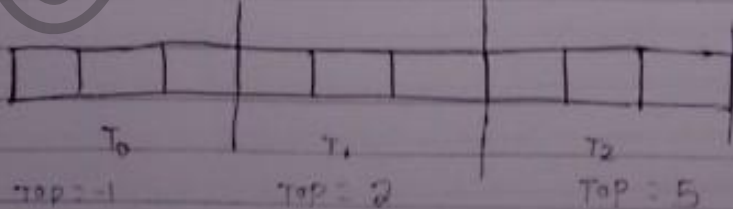
```

#

Implementing multiple Stack in arrays

$N=9$

$M=3$ (No. of Multiple stack)



Find top: 1st Stack: (how many stack) $\frac{N}{M} - 1$

$$= 9 \div 3 - 1 = 2$$

$$1^{st} \text{ stack} = (1 \times \frac{9}{3}) - 1 = 3 - 1 = 2$$

$$2^{nd} \text{ stack} = 2(\frac{9}{3}) - 1 = 2 \times 3 - 1 = 5$$

$$\vdots$$

$$50^{th} \text{ stack} = 50(\frac{9}{3}) - 1 =$$

$$i^{th} \text{ stack} = \boxed{i(\frac{M}{H}) - 1}$$

Top

When we say the Top is full:

Ans) When top of stack of Top is equal to Top of 1st stack.

When 50th stack is full

$$n) \quad \text{Top}_{50^{th}} = 51(\frac{9}{3}) - 1 \quad (\text{next stack top})$$

push opⁿ

push (S, N, H, Ti, x) (Ti = Top of ith stack)

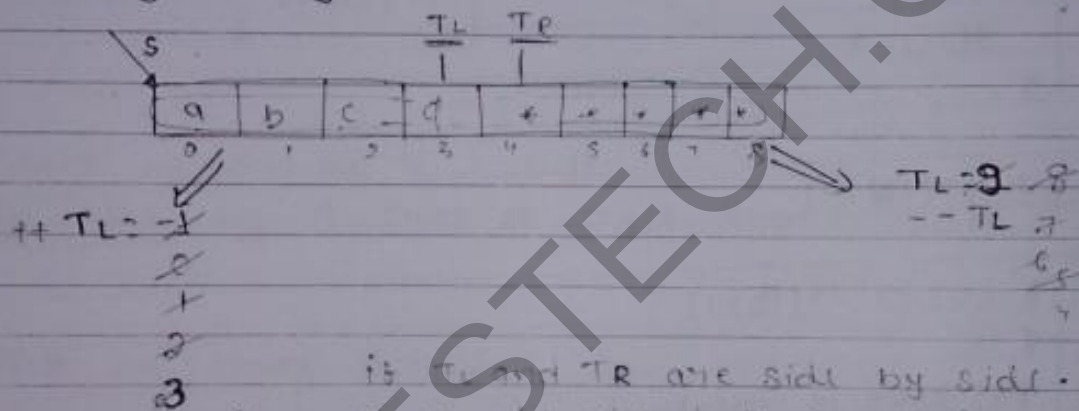
```
{
  if (Ti == (i+1)(N/M)-1)
  {
    printf("stack is full");
    exit(1);
  }
  else
  {
    Ti++;
    S[Ti] = x;
  }
}
```

pop (S, Ti) N, M)

```
{
  if (Ti == i(N/M)-1)
  {
    printf("stack is underflow");
    exit(1);
  }
  else
  {
    x = S[Ti];
    Ti--;
    return x;
  }
}
```

4) using above program we can implement multiple stack in single array but it is not efficient if '1' stack is full other remaining stack are empty if it giving stack is overflow.

4) Efficient way of implementing multiple stack in single array.



if TL and TR are side by side. then stack will give full condition but efficient way.

4) $TL = TR - 1$

both should side by side
 $TL = 5$ $TR = 6$

$TL = TR - 1$

Application of stack

- 1) Recursion
 - a) total recursion
 - b)

1) Recursion

- i) Tail recursion
- ii) Non-tail recursion
- iii) Nested recursion
- iv) Indirect recursion

2) Infix to postfix

3) postfix evaluation

4) Tower of Hanoi

5) Fibonacci Series

Q1 Write a C program to find min^m element in the given array

Ans)

```
void min(int a[], int n)
{
    int i;
    a[0] = min;
    for (i = 1; i < n; i++)
        if (a[i] < a[0])
            min = a[i];
}
```

Recursive

```
// min(a, i, j)
```

```
{ if (i == j)
```

```
    return a[i];
```

```
else
```

```
{ if (a[i] > a[j])
```

```
    min(a, i+1, j);
```

```
else
```

```
    min(a, i, j-1);
```

```
}
```

Time complexity = $O(n)$

Space complexity = $O(n)$

Recursion

$T = 1 + T(n-1)$

2) $1 + T(n-1) = \boxed{O(1)}$

Q1 Write a C-program to print the given array in the reverse order.

A-

```
pr(a, i, j)
{
    if (i > j)
        return a[i];
    else
    {
        pr(a, i, j-1);
        printf("%d ", a[j]);
    }
}
```

$$T(n) = 1 + (n-1)$$

$$= O(n)$$

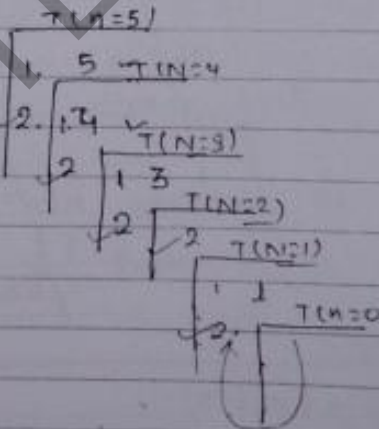
Space complexity: $O(n)$;

Time complexity: $O(n)$;

Tail recursion

##

```
TY(n)
{
    if (n <= 0) return 0;
    else
    {
        print(TY(n));
        TY(n-1);
    }
}
```



1. In the given recursive program, at the recursive function call is there then the function is called tail-recursive program.

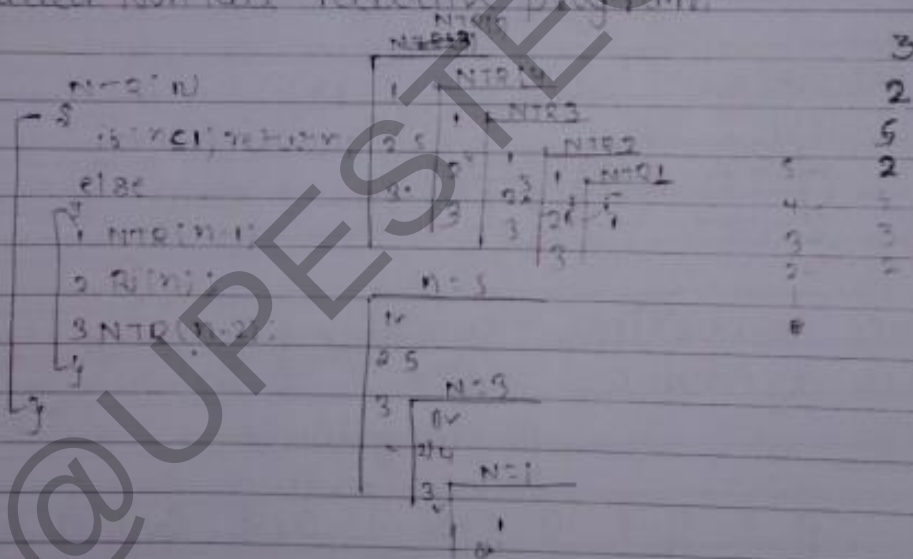
2. The draw back with tail recursive program is - we are waiting lot at stack step.

3. The advantage with the tail recursive program is, it is very easier to write, equivalent non recursive program with the help of for loop.

4. In given recursive program only one recursive program only one recursive it is also at the end then called tail recursive program.

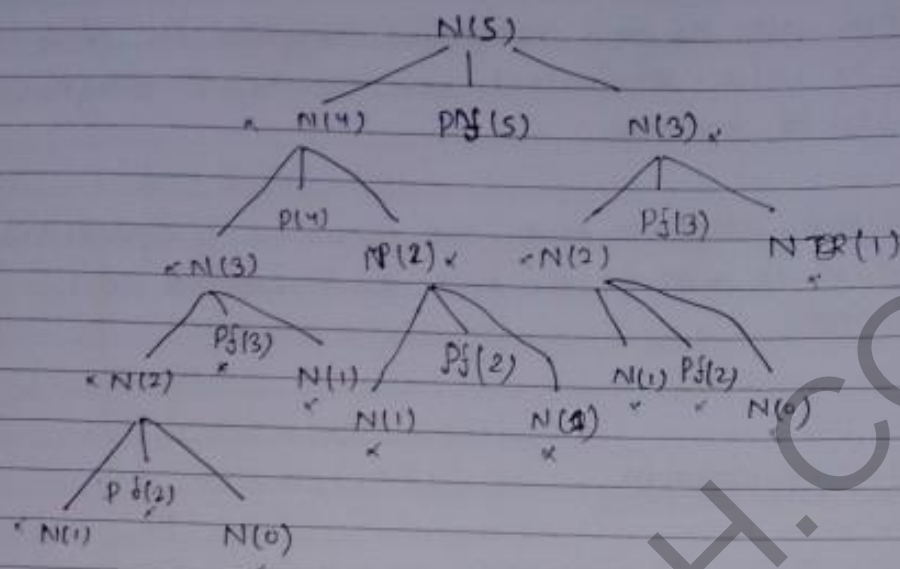
Non tail recursive program

In the given recursive program, After the recursive call there are something to do then recursive program called Non tail recursive program.



2 3 4 2 5 2 3 10

TREE METHOD



n level
binary
tree.
↓
 2^n
↓
 $O(2^n)$

2 3 4 2 5 2 3 1

T(n) = T(n-1) + T(n-2) + 1

T(n) = [with dynamic]

Space complexity: Table size + Stack

with dynamic ↓

How many distinct
function call

N + N

$$S = O(2N)$$

without dynamic $T = O(2^n)$

Space: $O(n)$

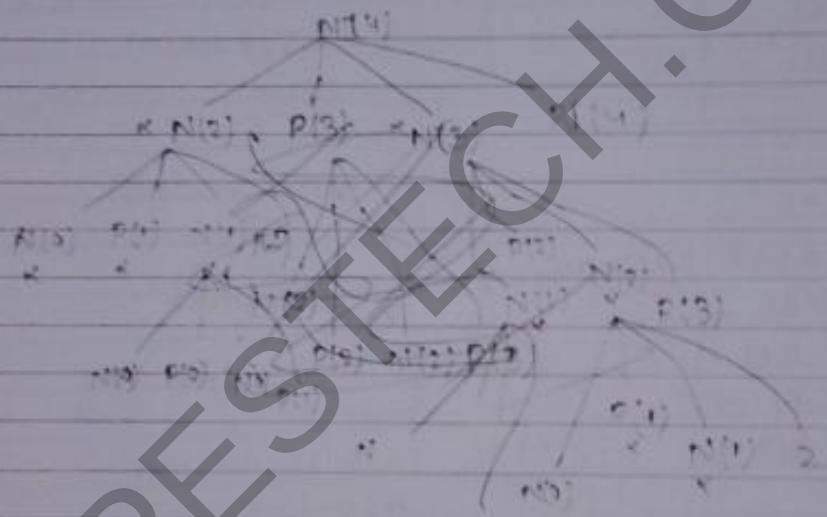
with dynamic $T = O(n)$

2) Consider the recursive program

```

NTR(n)
{
    if (n <= 0) return
    else
    {
        NTR(n-2)
        P(n-1)
        NTR(n-1)
        P(n)
    }
}
    
```

NTR(4)



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

1 0 1 2 3 0 1 2 1 0 1 2 3 4

Time complexity $\Rightarrow O(n)$ } dynamic programming
 Space complexity $\Rightarrow O(n)$

when we function store in table then it dynamic programming

NOTE! We are not wanting stack space necessarily
⇒ It is very difficult to find non-recursive program with
the help of for loop.

⇒

Indirect recursion

```
#1 A() B()
   { B() } A()
   }
```

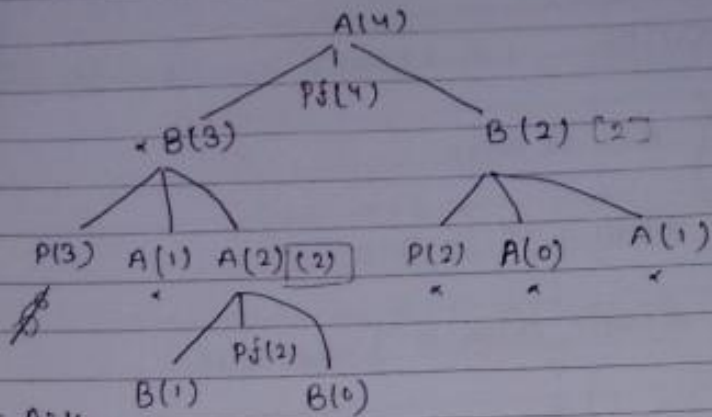
T/P = A(4)

```
A(n) B(n)
{ if (n ≤ 1) return;
  else
  { B(n-1);
    pf(n);
    B(n-2);
  }
}

{ if (n ≤ 1) return;
  else
  { pf(n);
    A(n-2);
    A(n-1);
  }
}
```


1 2 3 4

Date: _____



IP = A = 4

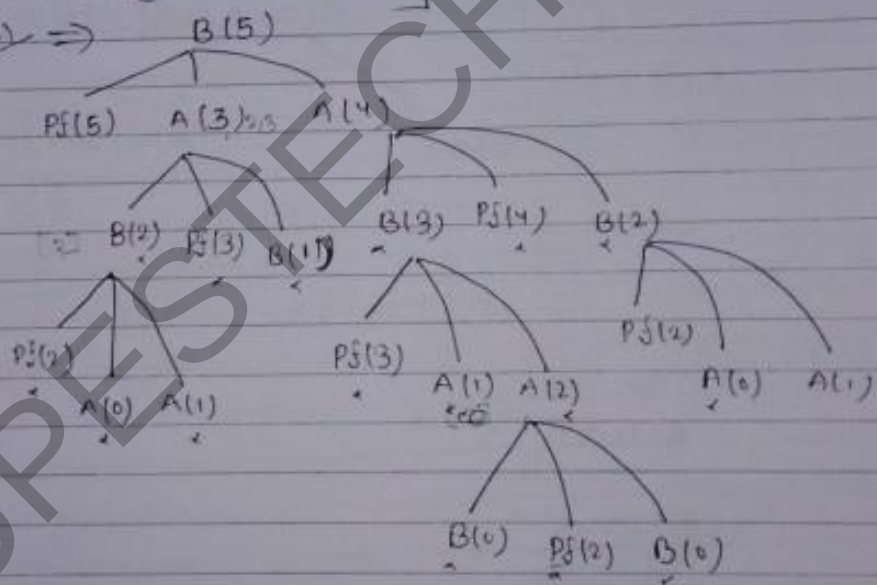
⇓

3 2 4 2 Ans.

[(P = B = 5)]

⇓

(ans at last) ⇒



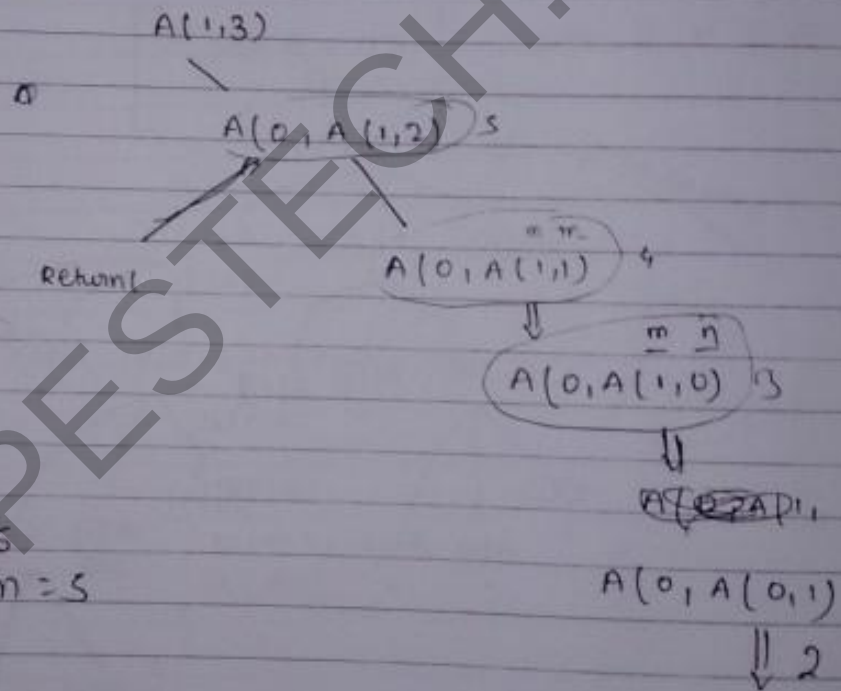
5 2 3 3 2 4 2

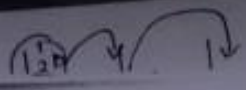
If $A(1) = 1$ then distance ~~function~~ function = n^2
 dist

Nested recursion (Ackermann's recursion relation)

$$A(m, n) = \begin{cases} n+1 & \text{if } m=0 \\ A(m-1, 1) & \text{if } n=0 \\ A(m-1, A(m, n-1)) & \text{otherwise} \end{cases}$$

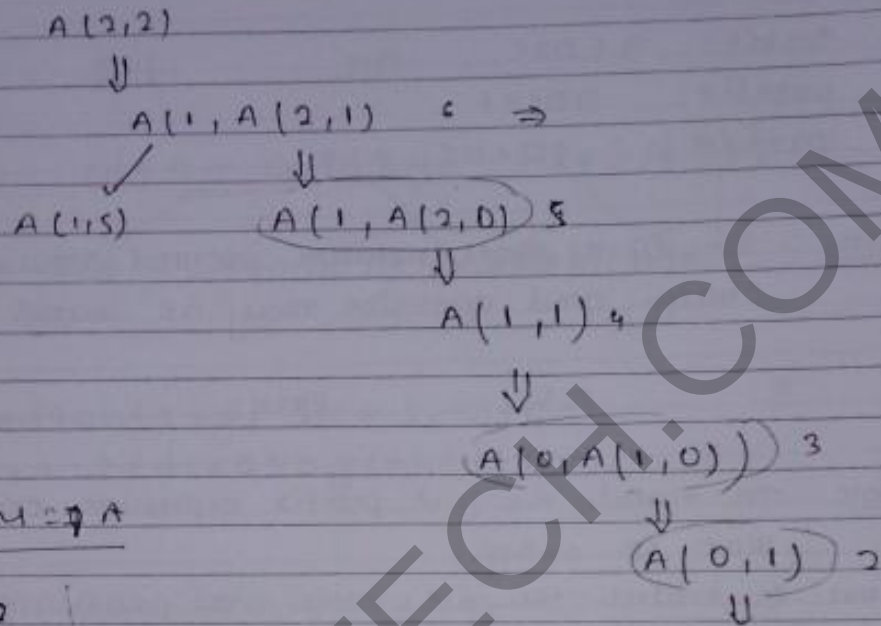
$A(1, 3)$





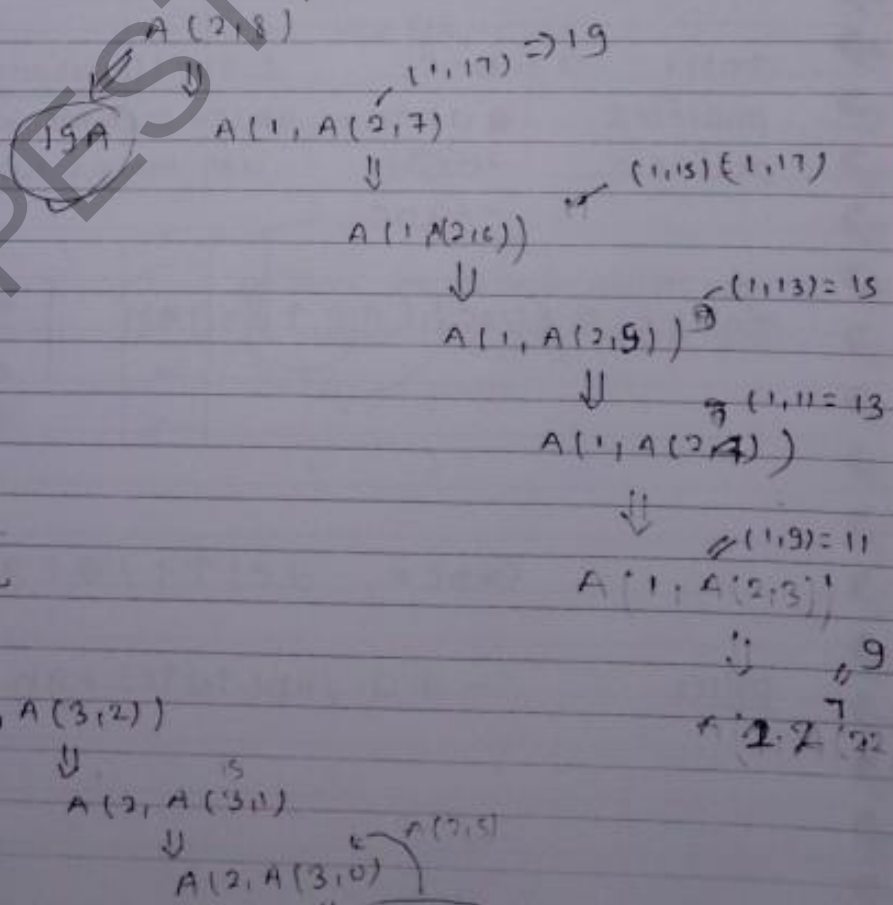
Date: _____

$A(2,2)$



Ans: 9

- $(1,0)=2$
- $(1,1)=3$
- $(1,2)=4$
- $(1,3)=5$
- $(1,4)=5$
- $(1,5)=7$
- $(1,6)=8$



$A(3,3)$



GI AN

$A(3,3)$



$A(2, A(3,2))$



$A(2, A(3,0))$



$A(2, A(3,0))$

Infix to postfix

Infix: $a + b * c$

postfix: $abc * +$

prefix: $+ a * bc$

Note :- In all the three notation operand order can't be change and operator may be change.

Infix Post Prefix

Note To evaluate the give postfix expression exactly one scan is enough.

but to evaluate the give infix and prefix many scan are required.

Infix: $a + b - c$ ($+$, $-$) ((priority same))

postfix: $abc - +$ when same who is in first with

prefix: $+ - abc$ but having high priority.

$- + abc$

Infix: $(a + b * c) / d \uparrow e \uparrow f - g * h$

$\uparrow$ = higher priority

Associative property

for $\uparrow$ is right to left

$a + b * c \quad d e f \uparrow \uparrow / + g h * A$

prefix

$- + a / * b c \uparrow d \uparrow e * g h \quad \underline{A_1}$

12/11/19

Q1) infix = $e \uparrow d - a * b \uparrow f / g + h * c / i + j - k$

postfix = $ed \uparrow a - ab \uparrow f / g + h * c / i + j - k$

$ed \uparrow a - a * b \uparrow f / g + h * c / i + j - k$

prefix = $- + - \uparrow ed / * a \uparrow b f / g + h * c / i + j - k$

postfix = $ed \uparrow a b \uparrow f \uparrow + g / - h * c / i \uparrow + j + k -$

prefix = $- + + - \uparrow ed / * a \uparrow b g / + h c / j + k$

Bracket Associativity is $L \rightarrow R$

$a \uparrow b * c$

Post Fix

$a b c + +$

+	3 pop
+	2

$a + b * c \uparrow d - e$

↑
↑
-

When we occur with $\uparrow$ lower priority than pop higher prior

$a b c d \uparrow + + e -$

-

a ↑ b ↑ c ↑ d ↑ e

a b c d e ↑ ↑ ↑ ↑ ↑

⊙
↑
↑
↑
↑

- # Lower priority is coming then push
- # High to low ⇒ pop
- # Left to right ⇒ pop
- # Right to left ⇒ push

Infix e ↑ d - a * b ↑ f / g + h * c / i + j - k

e d ↑ a b f ↑ * g / - h c + i / ⊙ + j + ⊙ k -

While converting infix to postfix we are using operator stack (only operators are push into stack)

↑
↑
↑

max^m stack space is = '3'.

a + b ↑ (c + (d * e) ↑ f) ↑ g

a b c d e f ↑ ↑ g ↑ ↑ +

*
(
+
(
↑
+

prefix to postfix

	prefix		postfix
1)	a	—	a
2)	+ab	—	ab+
3)	*a+bc	—	abc+*

4) $+ * / - a b c d \uparrow e / - e f g h$

$ab-c/d * ef-gh \uparrow +$

⇒

$* a / b - c \uparrow \uparrow \uparrow g f h + e * f, g, \text{ (Try at home)}$

$g a b / c g f h \uparrow \uparrow e f g, * + \uparrow \text{ (b d g) } = / *$

Another way Start from right.

*B

$+ a / b - c \uparrow \uparrow \uparrow g f h + e * f, g,$

#			$14 + 2 = 16$
	6	*	$pop = OP_2 = 6 \quad OP_1 = 3 \quad = 18$
2	3 18	-	$pop = OP_2 = 18 \quad OP_1 = 14$
14	16		

$16 - 18 = -2 \quad \underline{A}$

```

# int ois;
  x = postfix[i]
  write ( x == 10 )
$
  if ( x is operand )
    $ push(x);
    i++
  }
else
  {
    OP2 = pop();
    OP1 = pop();
    r = OP2 * OP1;
    push(r);
  }

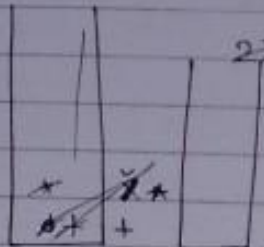
```

$\Theta(n)$ = time complexity
operand stack
Max^m size = $\lceil \frac{n}{2} \rceil$
Stack

NOTE While evaluating the given postfix expression we are using operand stack. only operand are pushed into stack.

infix = $2 + 3 * 4 - 5 + 6 / 7 * 4 + 5$

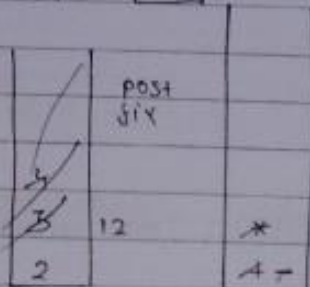
postfix



$2\ 3\ 4\ * - 5\ 6\ 7\ /\ 4\ * + 5$

postfix = $2\ 3\ 4\ * + 5 - 6\ 7\ /\ 4\ 5\ + * +$

evaluation $\rightarrow$



[Max Stack size = '2']

Fibonacci Series

n	0	1	2	3	4	5	6	7	8
fib(n)	0	1	1	2	3	5	8	13	21

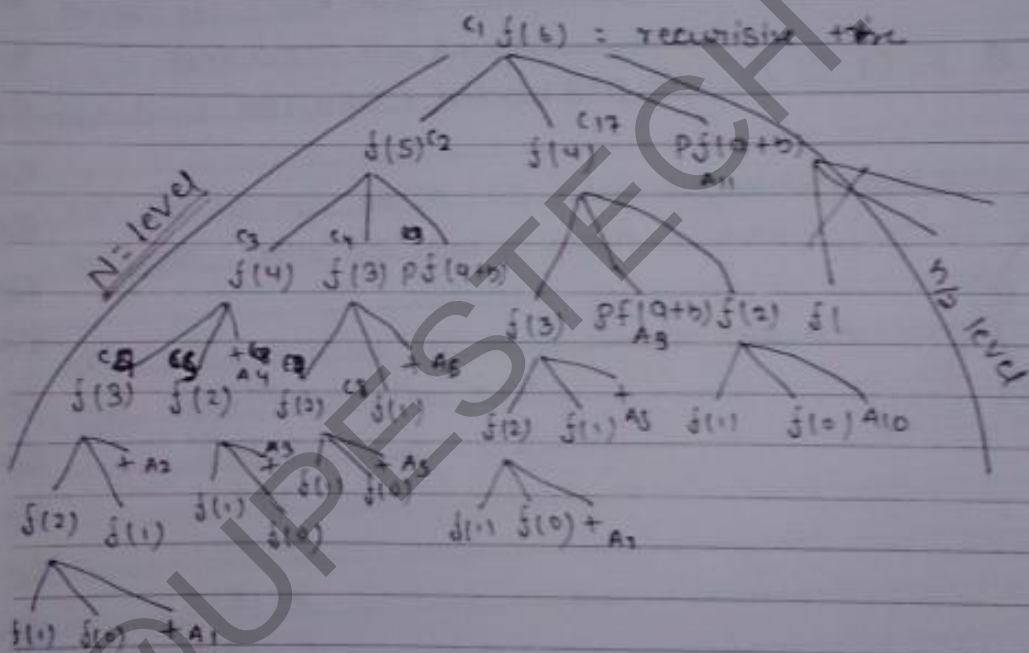
$$fib(n) = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

combinatorics = n if $n=0$ and $n=1$

```

fib(n)
{
    if (n==0 || n==1)
        return n;
    else
    {
        a = fib(n-1);
        b = fib(n-2);
        return a+b;
    }
}

```

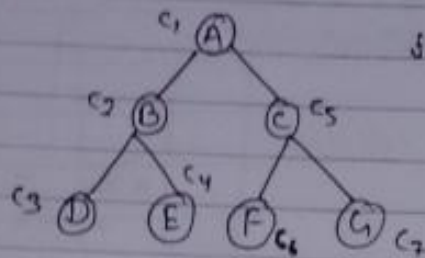


Make tree and give function name

Max^m Stack Size No. of labels

= 2ⁿ - 1

Function called in programming language = preorder.



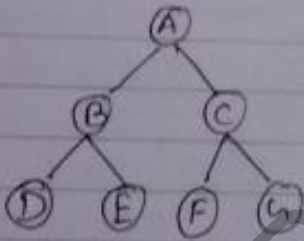
function call sequence.

A B D E C F G = function call sequence $\Rightarrow$ preorder

function Evaluation

D E B F G C A $\Rightarrow$ post order (function evaluation)

B.



Q1) In fibonacci of 'N' after how many function first addition will perform = $N+1$ function call.

Ans) Draw recursive tree it may be for any recursive program and give fun^c call order and count.

Q2) In fibonacci of 'n' is there any addition after last function call.

Ans) yes (and also $N/2$ addition may be there)
After Q5 = A_{10}, A_{11}, A_{12} .

Q3) In fibonacci of 'n' is there any function call after last addition.
Ans) No after A_{12} total program is over.

Q4) In fibonacci of 'N' how many function call are there.

Ans) Max^m function $O(2^n)$

$$f(0) = 1$$

$$f(1) = 2$$

$$f(2) = 3$$

$$f(3) = 5$$

$$f(4) = 9$$

$$f(5) = 15$$

$$f(6) = 25$$

$$f(7) = 41$$

:

$$f(n) = f(n-1) + f(n-2) + 1$$

In Fibonacci of 'n' how many addition will be there.
No. of addition

$$\Rightarrow f(0) = 0;$$

$$\Rightarrow f(1) = 0;$$

$$\Rightarrow f(2) = 1$$

$$T(n) = 1 + T(n-1) + T(n-2)$$

$$f(3) = 2$$

$$f(4) = 4$$

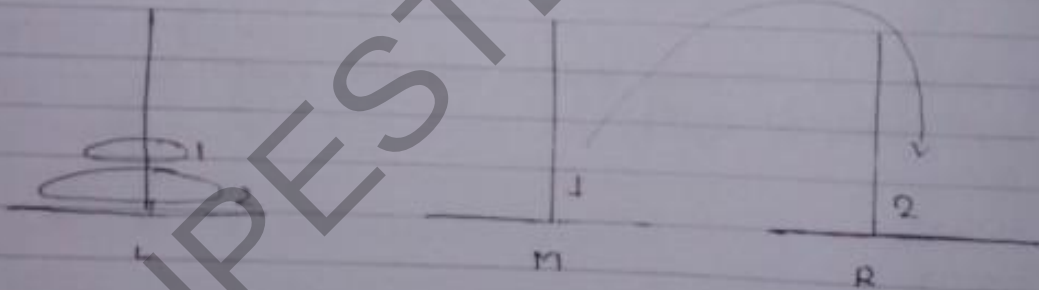
$$f(5) = 7$$

$$f(6) = 12$$

$$f(7) = 20$$

$$f(n) = f(n-1) + f(n-2) + 1$$

Tower of Hanoi

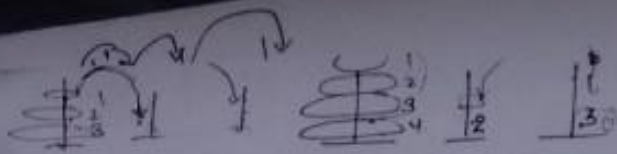


Move:

1) L - M

2) L - R

3) M - R



N=3

L-R

L-H

R-H

L-R

H-L

H-R

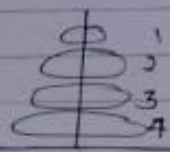
L-R

N=4

L-R

L-H

R-H



N=4 $\Rightarrow$

Top 3 have to send Middle = ie take 7 Step.

the move L-R (4 No block)

then 3 block at middle take 7 step max total step '7'.

1) L-H

6) R-H

11) R-L

2) L-R

7) L-H

12) H-R

3) H-R

8) L-R

13) L-H

4) L-H

9) H-R

14) L-R

5) R-L

10) H-L

15) H-R

N=5

1) Move Top 4 from L to M = 15

2) Move Top 1 from L to R = 1

3) Move Top 4 from M to R = 15

} = 31 Step.

$T_{OH}(n, \underset{\text{Source}}{L}, \overset{\text{Destination}}{R}, M)$

{
If $(n == 0)$ return

else

{
 $T_{OH}(n-1, \underset{S}{L}, \underset{D}{R}, M)$

 // If $n=1$:

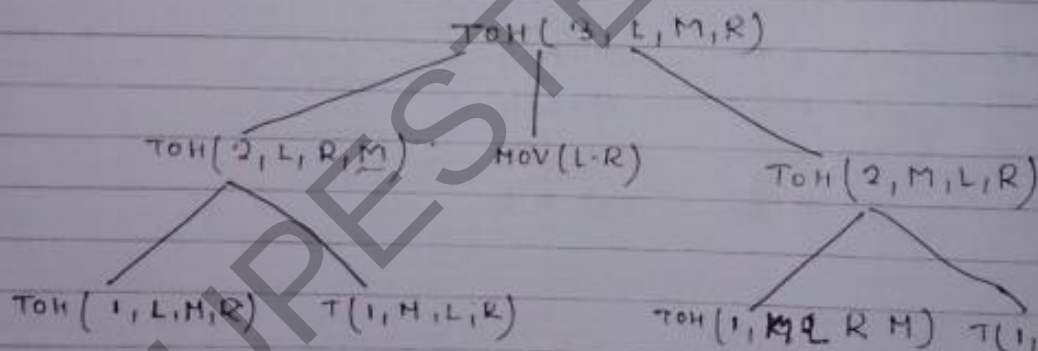
 Move $(L \rightarrow R)$

$T_{OH}(n-1, M, L, R)$

$$T(n) = T(n-1) + 1 + T(n-1)$$

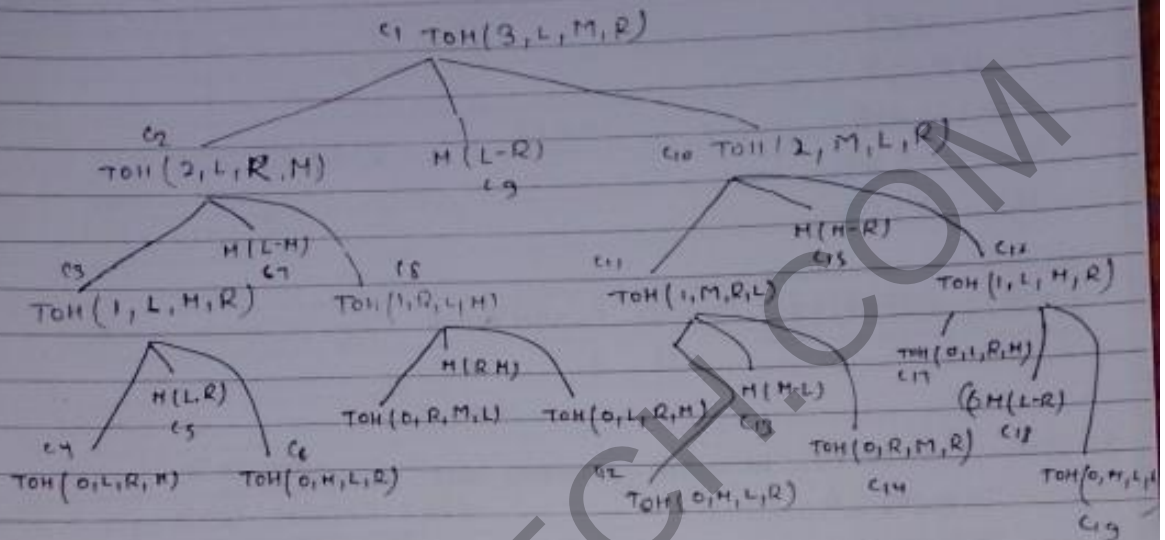
$$T(n) = 2T(n-1) + 1 \text{ (No. of moves)}$$

$$T(n) = 2T(n-1) + C = \text{time complexity}$$





Recursive + TC for TOH



Q1 In TOH(n) after how many function call move is take place

Ans) $N+1$ function + [After C4 there is H.]

Q2 TOH(n) is there any move after last function call?

Ans) No move (last)

Q3 In TOH is there any function call after last move

→ after H there is [C15 (last function call)]

Q4 In TOH of '3' after how many function there is move for M

M → R

Ans) It is 6th move - (before it C12 function call)

Q5 In TOH of 'N' how many function call are there

No. of function call

$$T_{OH}(0) = 1$$

$$T_{OH}(1) = 3$$

$$T_{OH}(2) = 7$$

$$T_{OH}(3) = 15$$

$$T_{OH}(4) = 31$$

$$T_{OH}(5) = 63$$

$$T(n) = 2T(n-1) + 1$$

Q1. In T_{OH} of 'n' how many nodes are there :

$$T_{OH}(0) = 0$$

$$T_{OH}(1) = 1$$

$$T_{OH}(2) = 3$$

$$T_{OH}(3) = 7$$

$$T_{OH}(4) = 15$$

$$T_{OH}(5) = 31$$

$$T(n) = (2T(n-1) + 1)$$

$$T(n) = 2^n - 1$$

Time $T(n) = O(2^n)$

Space: No. of level $O(n)$

Dynamic programming

$$T(n) = \dots$$

$$(n+1) + (3) + (2) + (1) = 6(n+1)$$

$$T = O(n)$$

Space complexity = stacksize + auxiliary function

$$n + 6n$$

$$= 7n$$

Chapter

Q5) C - Row-Major Order

char a[100][100]

Base address = '0'

$$a[40][50] = 0 + (40)100 + (50-0)$$

$$= 4000 + 50$$

$$= 4050 \text{ A}$$

06) a 7) a 8) c 9) a 10) b 11) a 12) c 13) 14) 15) d

Q8)

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

Q9)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

11) int a=10;
int b=a

Q10)

1	5	9	13
0	0	10	14
0	0	11	15
0	0	0	16